

## 7. Natural Convection in Electronic Devices

### 7.1 Introduction

Natural or free convection occurs due to the change in density of the fluid caused by heating process. In a gravity field, the fluid, which has a lower density, is lighter and therefore rises, creating a movement in the fluid which is called convection. This movement permits the fluid to pick up heat and carry it away.

The natural convection is the common method used in electronics cooling; there is a large class of equipment that lends itself to natural convection. This category includes stand-alone packages such as modems and small computers having an array of printed circuit boards (PCB) mounted within an enclosure.

The general liquids coolant use in electronics cooling as in the case of air, and liquid cooled radar transmitter and test equipment fluorinert liquid (FC-77).

The general equation to define the convective heat transfer either forced or free is given by the Newton's law of cooling:

$$q = hA(T_s - T_\infty) \quad (7.1)$$

The convection heat transfer coefficient (h) is expressed by the dimensionless Nusselt number (Nu) which is related to the dimensionless ratios Grashof (Gr) and Prandtl (Pr) numbers or the Rayleigh number (Ra) which is the product of the last two dimensionless groups.

$$Nu = Nu(Gr, Pr) = \frac{hL}{k} \quad (7.2)$$

Where:

$$L = \text{characteristic length (m)} = \frac{\text{Surface area (A)}}{\text{perimeter of the surface (P)}}$$

k = Thermal conductivity of the fluid (W/m.°C)

And

$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \quad (7.3)$$

Where:

g = Gravitational acceleration = 9.81 m/s<sup>2</sup>

β = Volume coefficient of expansion = 1/T (for ideal gas), T is the absolute temperature

L = Characteristic length (m)

ν = Kinematics viscosity (m<sup>2</sup>/s)

and

$$Pr = \frac{\mu C}{k} \quad (7.4)$$

Where:

μ = Dynamic viscosity (kg/m.s)

C = Specific heat (J/kg.°C)

$k$  = Thermal conductivity of the fluid (W/m.°C)

## 7.2 Empirical Correlations for Free Convection

All the average free-convection heat-transfer coefficients for external flow can be summarized in the following expression

$$\begin{aligned} \overline{Nu} &= \frac{\overline{h}L}{k} = c(\text{Gr Pr})^m \\ &= c(Ra)^m \end{aligned} \quad (7.5)$$

The constants  $c$ ,  $m$  are given in Table 7.1 for the uniform surface temperature case. The fluid properties are evaluated at mean film temperature ( $T_f$ ) where  $T_f = (T_s + T_\infty)/2$ .

**Table 7.1 Constants for Equation 7.5 for isothermal surfaces**

Geometry	Ra	c	m
Vertical planes and cylinders	$10^{-1}$ - $10^4$	use Figure 7.4	use Figure 7.4
	$10^4$ - $10^9$	0.59	0.25
	$10^9$ - $10^{13}$	0.1	1/3
Horizontal cylinder	$0$ - $10^{-5}$	0.4	0
	$10^{-5}$ - $10^4$	use Figure 7.5	use Figure 7.5
	$10^4$ - $10^9$	0.53	0.25
	$10^9$ - $10^{12}$	0.13	1/3
	$10^{-10}$ - $10^{-2}$	0.675	0.058
	$10^{-2}$ - $10^2$	1.02	0.148
	$10^2$ - $10^4$	0.85	0.188
	$10^4$ - $10^7$	0.48	0.25
	$10^7$ - $10^{12}$	0.125	1/3
Upper surface of heated plates or lower surface of cooled plates	$2 \times 10^4$ - $8 \times 10^6$	0.54	0.25
Upper surface of heated plates or lower surface of cooled plates	$8 \times 10^6$ - $8 \times 10^{11}$	0.15	1/3
Lower surface of heated plates or upper surface of cooled plates	$10^5$ - $10^{11}$	0.27	0.25
Vertical cylinder, height = diameter Characteristic length = diameter	$10^4$ - $10^6$	0.775	0.21
Irregular solids Characteristic length = distance Of fluid particle travels in boundary layer	$10^4$ - $10^9$	0.52	0.25

The characteristic length for different geometries is:

- vertical plate                       $L = \text{height}$
- Horizontal plate                 $L = W/2$  ,  $W = \text{width}$
- Spheres                             $L = D$
- Horizontal tube                  $L = D$
- Vertical tube

If:

$$\frac{D}{L} \geq \frac{35}{\sqrt[4]{Gr_L}} \quad \therefore L = \text{length (L)}$$

If not;

$$\therefore L = D$$

### 7.2.1 Free Convection over Vertical Plates and Cylinders

Natural heat transfer from a vertical plate is shown in Figure 7.1.

**At Uniform Surface Temperature ( $T_s = \text{constant}$ )**

For wide ranges of the Rayleigh number we can use the correlations given by Churchill and Chu

$$\overline{Nu}_L = 0.825 + \frac{0.387Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}} \quad \text{For } 10^{-1} < Ra_L < 10^2 \quad (7.6)$$

This equation has better accuracy for laminar flow and the subscript L indicates the characteristic length based on plate height.

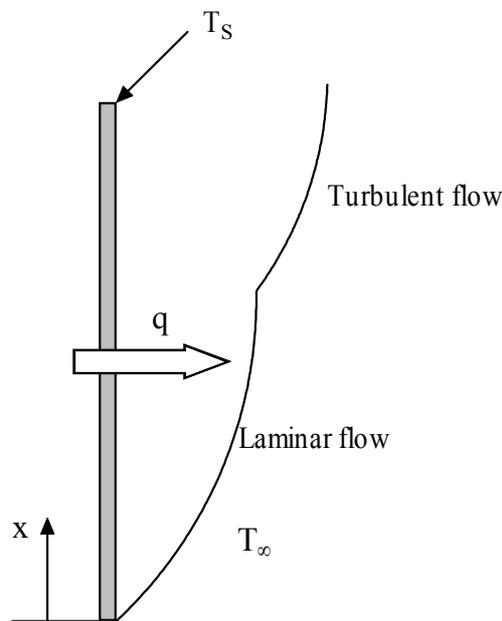


Figure 7.1 Flow over vertical plate

$$\overline{Nu}_L = 0.68 + \frac{0.67Ra_L^{1/4}}{\left[1 + (0.492/Pr)^{9/16}\right]^{4/9}} \quad \text{For } Ra_L < 10^9 \quad (7.7)$$

Fluid properties for both Equation 7.6 and 7.7 are evaluated at mean film temperature.

**For Uniform Surface Heat Flux ( $q'' = \text{constant}$ )**

If the plate surface has a constant heat flux; the Grashof number is modified as  $Gr^*$  based on  $x$ -direction as following:

$$Gr_x^* = Gr_x Nu_x = \frac{g\beta q'' x^4}{k\nu^2} \quad (7.8)$$

The local heat transfer coefficients given as follow

$$Nu_x = \frac{hx}{k} = 0.6(Gr_x^* Pr)^{1/5} \quad 10^5 < Gr_x^* < 10^{11} \quad (7.9)$$

$$Nu_x = \frac{hx}{k} = 0.17(Gr_x^* Pr)^{1/4} \quad 2 \times 10^{13} < Gr_x^* Pr < 10^{16} \quad (7.10)$$

Fluid properties for both Equations 7.9 and 7.10 are evaluated at local film temperature.

To get the average heat transfer coefficient for the constant heat flux integration along the plate height is performed.

$$\bar{h} = \frac{1}{L} \int_0^L h_x dx$$

Applying this integration to Equation 7.9 we get the average heat transfer coefficient in the form

$$\bar{h} = \frac{5}{4} h_{x=L}$$

**7.2.2 Free Convection from Horizontal Cylinder**

For flow over a horizontal cylinder with uniform surface temperature for wide ranges of Ra the following expressions are given by Churchill and Chu

$$\overline{Nu}_D^{1/2} = 0.6 + 0.387 \left( \frac{Gr Pr}{[1 + (0.559 / Pr)^{9/16}]^{16/9}} \right)^{1/6} \quad 10^{-5} < Gr Pr < 10^{12} \quad (7.11)$$

Again fluid properties for Equation 7.11 are evaluated at mean film temperature.

For horizontal cylinder with liquid metals following equation may be applied.

$$Nu_D = 0.53(Gr Pr^2)^{1/4} \quad (7.12)$$

**7.2.3 Free Convection over Horizontal Plates**

**For Uniform Surface Temperature ( $T_s = \text{constant}$ )**

We have many cases for horizontal plates with constant surface temperature as shown in Figure 7.2

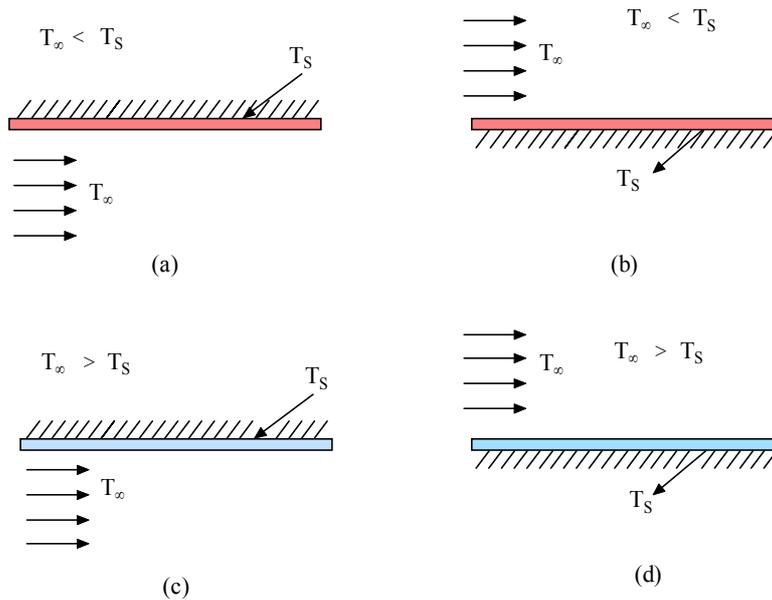


Figure 7.2 (a) Lower surface of heated plates, (b) Upper surface of heated plates  
(c) Lower surface of cooled plates, (d) Upper surface of cooled plates

For lower surface of heated plates or upper surface of cooled plates

$$\overline{Nu}_L = 0.27(Gr_L Pr)^{1/4} \quad 10^5 \leq Gr_L Pr \leq 10^{11} \quad (7.13)$$

For upper surface of heated plates or Lower surface of cooled plates the average heat transfer coefficient is expressed by:

$$\overline{Nu}_L = 0.54(Gr_L Pr)^{1/4} \quad 2 \times 10^4 \leq Gr_L Pr \leq 8 \times 10^6 \quad (7.14)$$

$$\overline{Nu}_L = 0.15(Gr_L Pr)^{1/3} \quad 8 \times 10^6 \leq Gr_L Pr \leq 10^{11} \quad (7.18)$$

Where fluid properties are evaluated at mean film temperature  $T_f = (T_s + T_\infty)/2$

**For Uniform Surface Heat Flux ( $q'' = \text{constant}$ )**

For hot surface facing upward (Upper surface of heated plate)

$$\overline{Nu}_L = 0.13(Gr_L Pr)^{1/3} \quad Gr_L Pr < 2 \times 10^8 \quad (7.19)$$

$$\overline{Nu}_L = 0.16(Gr_L Pr)^{1/3} \quad 2 \times 10^8 < Gr_L Pr < 10^{11} \quad (7.20)$$

For hot surface facing downward (Lower surface of heated plates)

$$\overline{Nu}_L = 0.58(Gr_L Pr)^{1/3} \quad 10^6 < Gr_L Pr < 10^{11} \quad (7.21)$$

For the Equations 7.19, 7.20, and 7.21 the fluid properties are evaluated at the equivalent temperature ( $T_e$ ) which is defined as

$$T_e = T_s - 0.25 (T_s - T_\infty)$$

Where:

$T_s$  = the average surface temperature

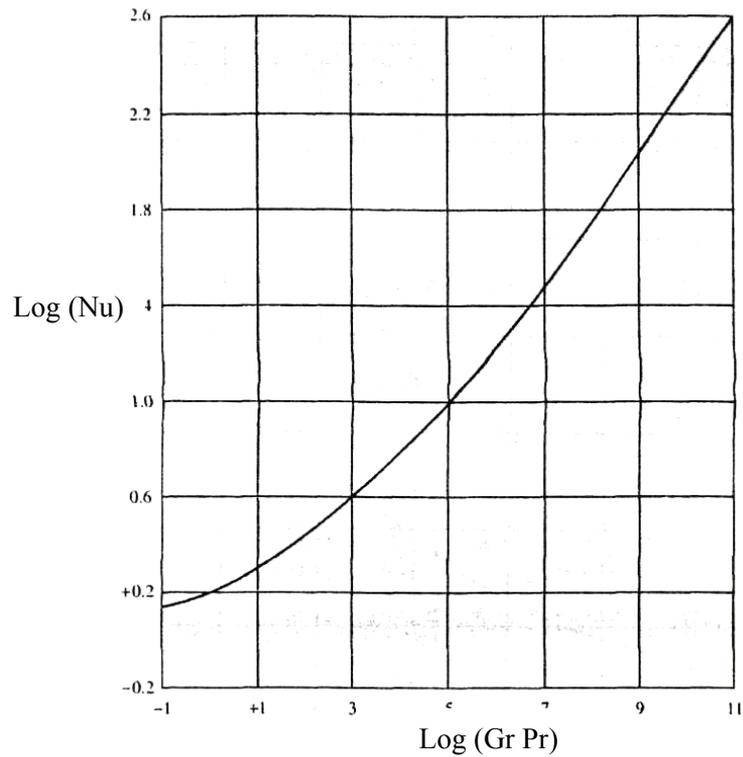


Figure 7.4 Heat transfer correlation for vertical planes and cylinders at ( $Ra = 10^{-1} - 10^4$ )

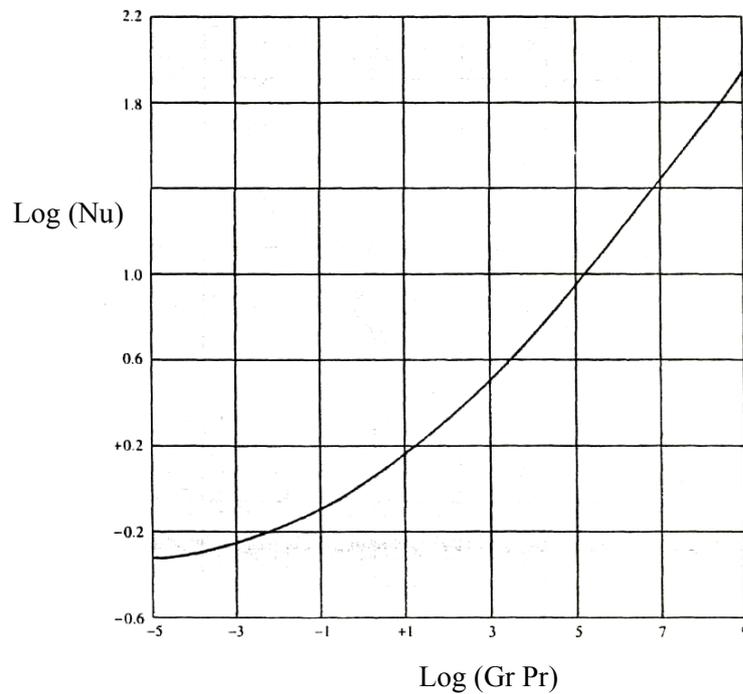


Figure 7.5 Heat transfer correlation for horizontal cylinder ( $Ra = 10^{-5} - 10^4$ )

### 7.2.4 Free Convection over Irregular Surfaces

For irregular surfaces the average heat transfer coefficient is given by Lienhard formula

$$\overline{Nu} = 0.52(Gr Pr)^{0.25} \quad (7.22)$$

Where fluid properties are evaluated at mean film temperature  $T_f = (T_s + T_\infty)/2$ . The characteristic length is the distance of a fluid particle travels in the boundary layer, for example, see figure 7.3, the characteristic length  $(L) = H + (w/2)$

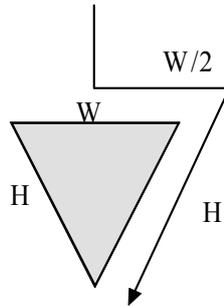


Figure 7.3 Travel distance of a particle along an irregular surface

### 7.2.5 Free Convection over Spheres

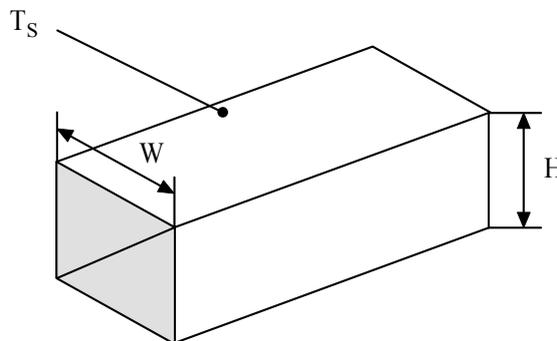
When the flow occurs on sphere the recommended correlation is provided by Churchill for  $Ra_D < 10^{11}$  and  $Pr > 0.5$

$$\overline{Nu}_D = 2 + \frac{0.589(Gr_D Pr)^{1/4}}{[1 + (0.469 / Pr)^{9/16}]^{4/9}} \quad (7.23)$$

Where fluid properties are evaluated at mean film temperature  $T_f = (T_s + T_\infty)/2$

**Example7.1:** Air flow across an electronic box used in a spacecraft that is 0.2 m high and 0.3 m wide to maintain the outer box surface at 45 °C. If the box is not insulated and exposed to air at 25 °C, Calculate is the heat loss from the duct per unit length.

Schematic:



**Solution:**

The properties of air evaluated at mean film temperature:

Part B: Heat Transfer Principals in Electronics Cooling

$$T_f = (45+25)/2 = 35^\circ\text{C}$$

Air properties are:

$$v = 16.7 \times 10^{-6} \text{ m}^2/\text{s}$$

$$k = 0.0269 \text{ W/m} \cdot ^\circ\text{C}$$

$$\text{Pr} = 0.706$$

$$\beta = 1/308 = 3.247 \times 10^{-3} \text{ K}^{-1}$$

For the two vertical sides:

The characteristic length is the height  $H = 0.2 \text{ m}$ .

The Gr Pr product is:

$$\text{Gr Pr} = \frac{9.81 \times 3.247 \times 10^{-3} \times 20 (0.2)^3}{(16.7 \times 10^{-6})^2} \times 0.706 = 12.9 \times 10^6$$

By using the Equation 7.7

$$\overline{Nu}_s = 0.68 + \frac{0.67 (12.9 \times 10^6)^{1/4}}{\left[1 + (0.492 / 0.706)^{9/16}\right]^{4/9}} = 31.47$$

The average heat transfer coefficient is

$$\overline{h}_s = \frac{\overline{Nu}_s k}{L} = \frac{31.47 \times 0.0269}{0.2} = 4.234 \text{ W/m}^2 \cdot ^\circ\text{C}$$

For the top surface (Upper surface of heated plates):

The characteristic length is half the width  $W/2 = 0.15 \text{ m}$ .

The Gr Pr product is:

$$\text{Gr Pr} = \frac{9.81 \times 3.247 \times 10^{-3} \times 20 (0.15)^3}{(16.7 \times 10^{-6})^2} \times 0.706 = 5.443 \times 10^6$$

By using the Equation 7.18: upper surface of heated plate case

$$\overline{Nu}_t = 0.15 (5.443 \times 10^6)^{1/3} = 26.38$$

The average heat transfer coefficient is

$$\overline{h}_t = \frac{\overline{Nu}_t k}{L} = \frac{26.38 \times 0.0269}{0.15} = 4.732 \text{ W/m}^2 \cdot ^\circ\text{C}$$

For the bottom (Lower surface of heated plates): The characteristic length half the width  $W/2 = 0.15 \text{ m}$ , same Gr Pr product

By using the Equation 7.13 lower surface of heated plate case

$$\overline{Nu}_b = 0.27 (5.443 \times 10^6)^{1/4} = 13.04$$

The average heat transfer coefficient is

$$\overline{h}_b = \frac{\overline{Nu}_b k}{L} = \frac{13.04 \times 0.0269}{0.15} = 2.34 \text{ W/m}^2 \cdot ^\circ\text{C}$$

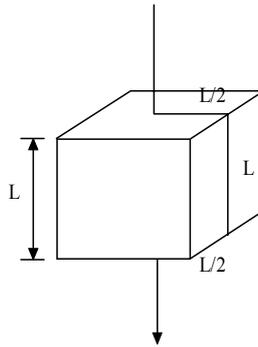
The heat loss from the electronic box per unit length is

$$q' = (T_S - T_\infty) [2 \times \bar{h}_S \times H + \bar{h}_t \times w + \bar{h}_b \times w]$$

$$= (45 - 25) [2 \times 4.234 \times 0.2 + 4.732 \times 0.3 + 2.34 \times 0.3] = 76.3 \text{ W/m}$$

**Example 7.2:** A cube of 10cm side length is left to cool in air. It is considered as irregular surface. The cube surface is maintained at 60 °C and exposed to atmospheric air at 10 °C. Calculate the heat transfer rate.

Schematic: to show the travel distance of a particle from the fluid along the surface



**Solution:**

The mean film temperature is:

$$T_f = (60+10)/2 = 35 \text{ }^\circ\text{C}$$

Thus air properties are

$$v = 17.47 \times 10^{-6} \text{ m}^2/\text{s}$$

$$k = 0.0268 \text{ W/m} \cdot \text{ }^\circ\text{C}$$

$$\text{Pr} = 0.7$$

$$\beta = 1/308 = 3.247 \times 10^{-3} \text{ K}^{-1}$$

From the schematic shown the characteristic length is

$$L = (L/2) + L + (L/2) = 2L = 20 \text{ cm}$$

The Gr Pr product is

$$\text{Gr Pr} = \frac{9.81 \times 3.247 \times 10^{-3} \times 50 (0.2)^3}{(17.47 \times 10^{-6})^2} \times 0.7 = 29.223 \times 10^6$$

By using the Equation 7.22

$$\bar{Nu} = 0.52 (29.223 \times 10^6)^{0.25}$$

$$= 38.23$$

The average heat transfer coefficient is

$$\bar{h} = \frac{\bar{Nu} k}{L} = \frac{38.23 \times 0.0268}{0.4} = 2.56 \text{ W/m}^2 \cdot \text{ }^\circ\text{C}$$

The total heat transfer surface area is

$$A = 6x (0.1 \times 0.1) = 0.06 \text{ m}^2$$

The total heat transfer is

$$q = 2.56 \times 0.06 \times 50 = 7.68 \text{ W}$$

### 7.3 Natural Convection from Finned Surfaces

The amount of heat that can be removed from an electronic component that is cooled by natural convection will be substantially increased if the surface area of the component can be substantially increased. One convenient method for increasing the surface area is to add fins as shown in Figure 7.6 with a low thermal resistance. The temperature of the fins will then be nearly equal to the surface temperature of the electronic component. The additional heat transfer to the atmosphere will be proportional to the increase in the surface area.

Fins will increase the size and weight of the electronic component. This may be a small penalty to pay if the cost is reduced and the reliability is increased by eliminating the need for a cooling fan.

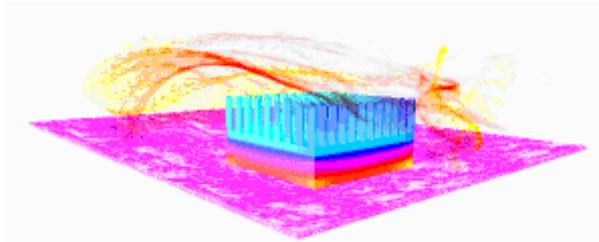


Figure 7.6 Finned surface over PCB to increase the heat transfer rate

The effectiveness of the finned surface will depend upon the temperature gradient along the fin as it extends from the surface of the electronic component. When the fin has a small temperature gradient, the temperature at the tip of the fin will be nearly equal to the temperature at the base of the fin or the chassis surface, and the fin will have a high efficiency.

Then the total heat transfer divided into two components due to the heat transferred from the free exposed surface of the electronic component and the heat transferred from fins surface as explained before is

$$\begin{aligned} q &= h [(A_{\text{tot}} - A_f) + \eta_f A_f] (t_s - t_\infty) \\ &= \eta_o A_{\text{tot}} h (t_s - t_\infty) \end{aligned} \quad (7.24)$$

Where:

$A_{\text{tot}} - A_f$  = free exposed surface area of the electronic component

$A_f$  = fins area

### 7.3.1 Natural Convection over PCBs

The natural convection cooling for PCBs are usually used when the heat loads are not too high. Where the PCBs are usually mounted within electronic chassis that are completely open at the top and bottom as shown in the Figure 7.7, and the minimum space between the components and the adjacent is 0.75 in to prevent choking of the natural convection flow.

Where the air enters at the bottom to remove the heating load from the PCBs, The warmer air has a reduced density, so that it starts to rise to finally exit at the top of the chassis. We must be neglect the radiation effect because the PCBs see each other. The heat flow through the PCB more easily than it flow over the external boundary layer due to a very low conduction thermal resistance through the PCB, compared with the convection thermal resistance over the external boundary layer on one face of the PCB.

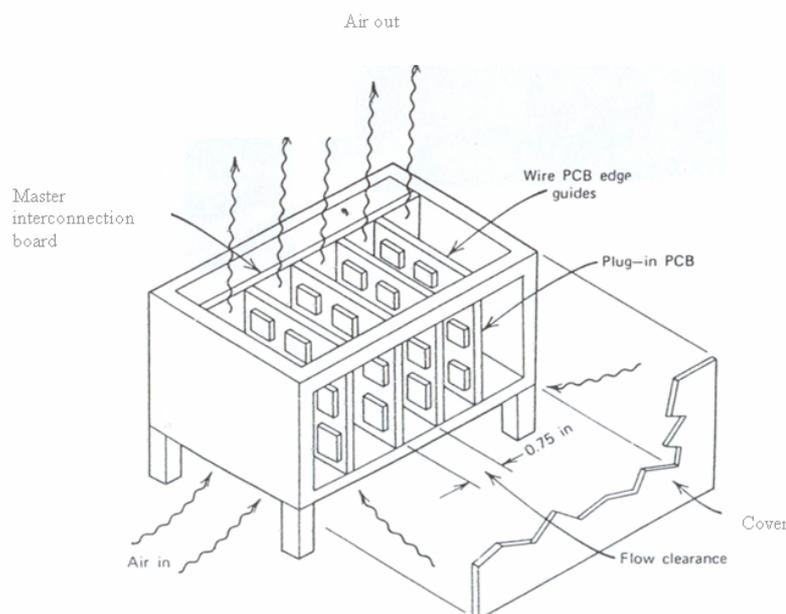


Figure 7.7 Example of natural convection heat transfer over PCBs

**Example 7.3:** a PCB inside electronic chassis in vertically position with 15 x 23 x 0.15 cm, the thermal conductivity for the PCB with ignoring lead wires is 2.25 W/m °C, and the convective heat transfer coefficient is 5 W/m<sup>2</sup> °C. Calculate the ratio between the convection thermal resistance to conduction thermal resistance.

**Solution:**

The convection thermal resistance is

$$R_{\text{convection}} = \frac{1}{hA} = \frac{1}{(5)(1.5 \times 2.3)} = 0.058 \text{ } ^\circ\text{C/W}$$

The conduction thermal resistance is

$$R_{\text{conduction}} = \frac{L}{KA} = \frac{0.015}{(2.25)(1.5 \times 2.3)} = 0.00193 \text{ } ^\circ\text{C/W}$$

The resistances ratio is

$$\frac{R_{\text{convection}}}{R_{\text{conduction}}} = 30$$

Comment: This example shows the convection thermal resistance is 30 times of the conduction thermal resistance through the PCB. So that the heat flow through the PCB more easily than it flow over the external boundary layer.

## 7.4 Natural Convection inside Enclosure

The free-convection flow phenomenon inside an enclosed space is one of the interesting examples of very complex fluid systems that may yield to analytical, empirical, and numerical solutions. Consider the system shown in Figure 7.8, where a fluid is contained between two vertical plates separated by the distance  $\delta$ . As a temperature difference  $\Delta T = T_1 - T_2$  is impressed on the fluid, a heat transfer will be experienced with the approximate flow regions shown in Figure 7.9, according to MacGregor and Emery. In this figure, the Grashof number is calculated as

$$Gr_{\delta} = \frac{g \beta \Delta T \delta^3}{\nu^2} \quad (7.25)$$

At very low Grashof number the heat transfer occurs mainly by conduction across the fluid layer. As the Grashof number is increased, different flow regimes are encountered, as shown, with a progressively increasing heat transfer as expressed through the Nusselt number

$$Nu_{\delta} = \frac{h \delta}{k} \quad (7.26)$$

The empirical correlations obtained were:

$$Nu_{\delta} = 0.42 (Gr_{\delta} Pr)^{1/4} Pr^{0.012} \left( \frac{L}{\delta} \right)^{-0.3} \quad q_w = \text{cons} \tan t \quad (7.27)$$

$$10^4 < Gr Pr < 10^7$$

$$1 < Pr < 20000$$

$$10 < L / \delta < 40$$

$$Nu_{\delta} = 0.46 (Gr_{\delta} Pr)^{1/3} \quad q_w = \text{cons} \tan t \quad (7.28)$$

$$10^6 < Gr Pr < 10^9$$

$$1 < Pr < 20$$

$$1 < L / \delta < 40$$

The heat flux is calculated as

$$\frac{q}{A} = q_w'' = h \Delta T \quad (7.29)$$

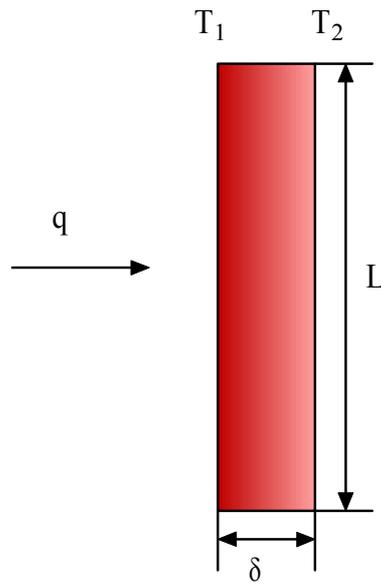


Figure 7.8 free convection inside vertical enclosure space

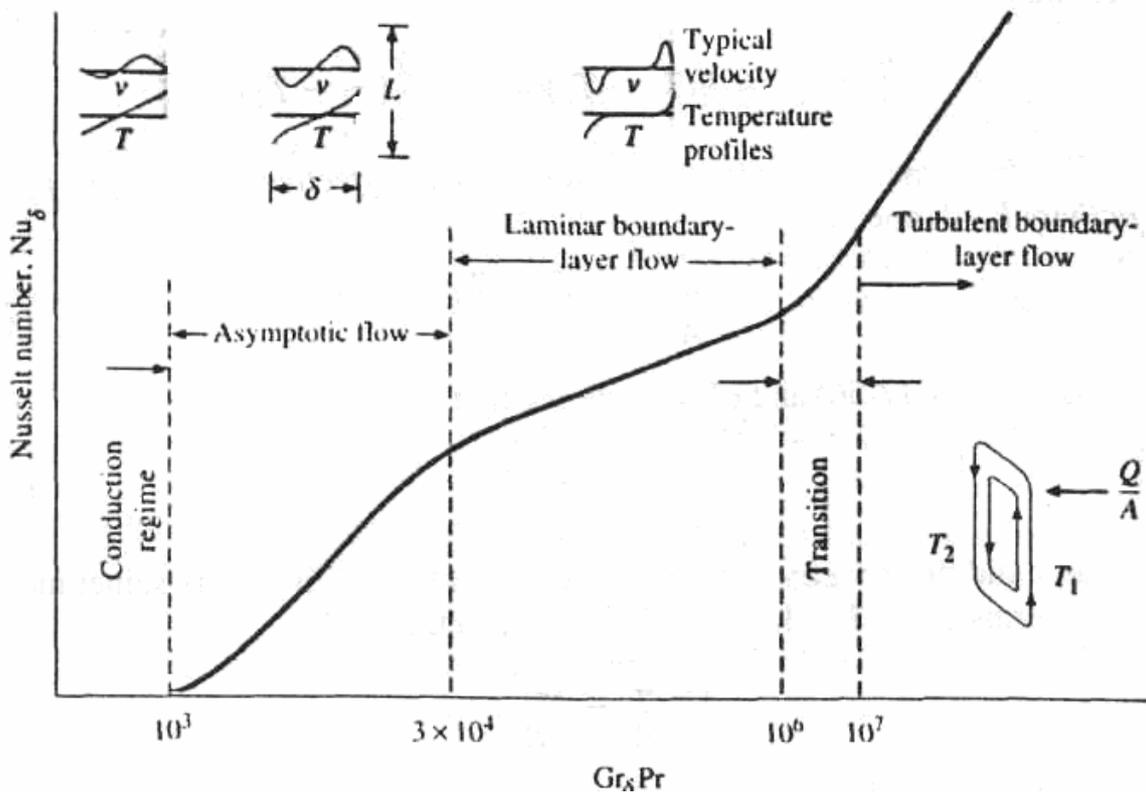


Figure 7.9 Flow regimes for the vertical convection layer inside enclosure