Part B: Heat Transfer Principals in Electronics Cooling

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4. Conduction Heat Transfer

4.1 Fourier Equation for Conduction

Conduction is one of the heat transfer modes. Concerning thermal design of electronic packages conduction is a very important factor in electronics cooling specially conduction in PCB's and chip packages. The basic law governing the heat transfer by conduction is Fourier's law (Equation 4.1).

$$q'' = -k\frac{dT}{dx} \tag{4.1}$$

The above equation is called the rate equation it calculates the heat transferred per unit area in the direction perpendicular to the area through which it's transferred.

4.2 General Governing Equation (Energy Equation)

As a system, energy balance may be applied on any electronic component. A typical energy balance on a control volume can be described as in Equation 4.2. The amount of energy flowing into or out of the system can be described by the Fourier's law.

$$\dot{E}_{in} + \dot{E}_{g} = \dot{E}_{out} + \dot{E}_{stored} \tag{4.2}$$

4.2.1 Cartesian Coordinates

A differential control volume is shown in Figure 4.1. The differential control volume has the side's dx, dy, dz respectively.

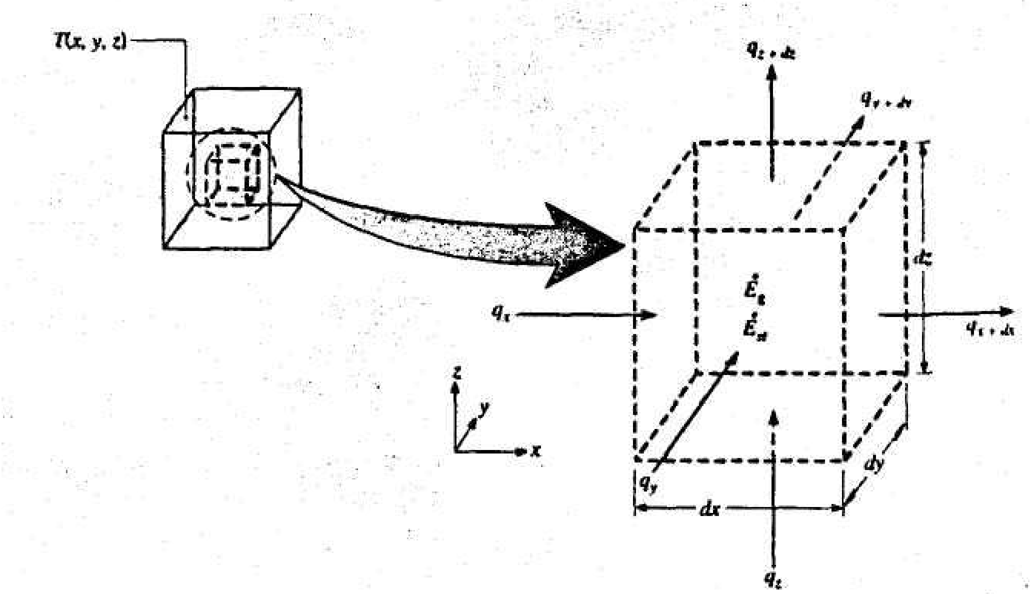


Figure 4.1 Differential control volume , dx dy dz, for conduction in cartesian coordinates.

From Fourier's law the heat flux (per unit volume) flowing into the control volume in the three Cartesian coordinates is

$$q''_x = -k\frac{\partial T}{\partial x} \qquad q''_y = -k\frac{\partial T}{\partial y} \qquad q''_z = -k\frac{\partial T}{\partial z} \tag{4.3}$$

Using Taylor expansion series and neglecting higher order terms, the surface flux for x, y and z after a displacement dx, dy and dz respectively could be expressed as

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x}dx \tag{4.4a}$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y}dy \tag{4.4b}$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z}dz \tag{4.4c}$$

In the above equations the rate of heat transferred at location x + dx equals that at location x in addition to the change of this rate with respect to x multiplied by the distance dx.

If there is a thermal energy generated by an energy source through the medium .This term is expressed as

$$\dot{E}_g = \dot{q}^{///}\, dx\, dy\, dz \tag{4.5}$$

Where, $\dot{q}^{///}$ is the rate at which energy is generated per unit volume of the medium in W/m^3.

If the material shows no change in phase, we may ignore the effects of latent energy, and the energy storage term may be expressed as

$$\dot{E}_{st} = \rho C_P \frac{\partial T}{\partial t} dx\, dy\, dz \tag{4.6}$$

Where, $\rho C_p dT/dt$ is the time rate of change of the sensible internal energy of the medium per unit volume.

Substituting from Equations 4.3, 4.5 and 4.6 in Equation 4.2 yields

$$q_x + q_y + q_z + \dot{q}^{///}\, dx\, dy\, dz - q_{x+dx} - q_{y+dy} - q_{z+dz} = \rho C_P \frac{\partial T}{\partial t} dx\, dy\, dz \tag{4.7}$$

Substituting from Equations 4.4, it follows that

$$-\frac{\partial q_x}{\partial x}dx - \frac{\partial q_y}{\partial y}dy - \frac{\partial q_z}{\partial z}dz + \dot{q}^{///}\, dx\, dy\, dz = \rho C_P \frac{\partial T}{\partial t} dx\, dy\, dz \tag{4.8}$$

Using Fourier's law, we obtain the conduction rates

$$q_x = -k\,dy\;dz\,\frac{\partial T}{\partial x} \tag{4.9a}$$

$$q_y = -k\,dx\;dz\,\frac{\partial T}{\partial y} \tag{4. 9b}$$

$$q_z = -k\,dx\;dy\,\frac{\partial T}{\partial z} \tag{4.9c}$$

Substituting Equations 4.9 into Equation 4.8 and dividing by the dimensions of the control volume (dx dy dz), we obtain

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}^{///} = \rho C_P\,\frac{\partial T}{\partial t} \tag{4.10}$$

For a constant thermal conductivity the heat equation could be rewritten as

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}^{///}}{k}=\frac{1}{\alpha}\,\frac{\partial T}{\partial t} \tag{4.11}$$

Where $\alpha = k/\rho\,C_p$ is the thermal diffusivity.

4.2.2 Cylindrical Coordinates

Similarly we can deduce the general heat equation in the cylindrical and spherical coordinates. The general equation in the cylindrical coordinates and the infinitesimal control volume on which the energy conservation is done is shown in Figure 4.2

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}^{///} = \rho C_P\,\frac{\partial T}{\partial t} \tag{4.12}$$

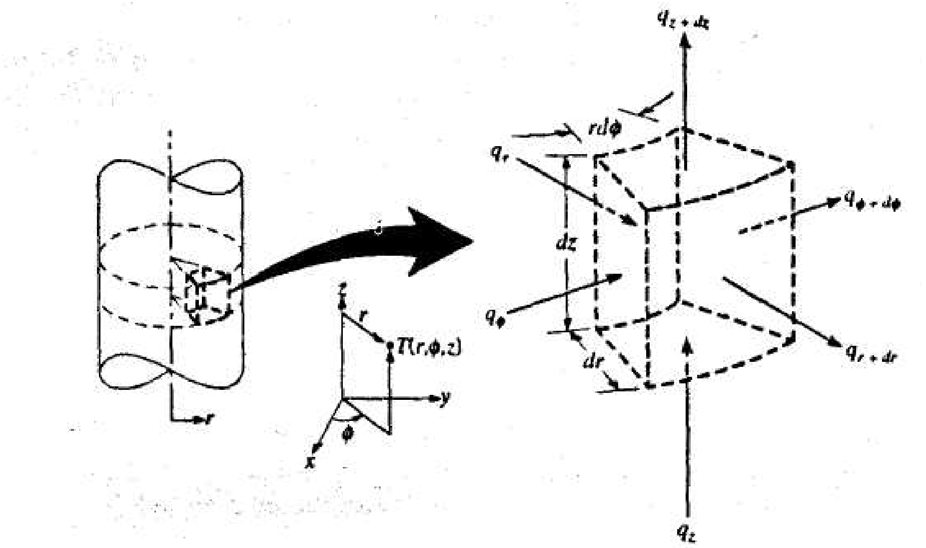


Figure 4.2 Differential control volume dr.rdϕ.dx for conduction analysis in cylinderical

coordinates (r, ϕ ,z)

4.2.3 Spherical Coordinates

For the spherical coordinates also the general form and the volume on which the energy conservation is done is shown in Figure 4.3

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(k r^2 \frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(k \sin\theta \frac{\partial T}{\partial z}\right) + \dot{q}^{///} = \rho C_P \frac{\partial T}{\partial t} \qquad (4.13)$$

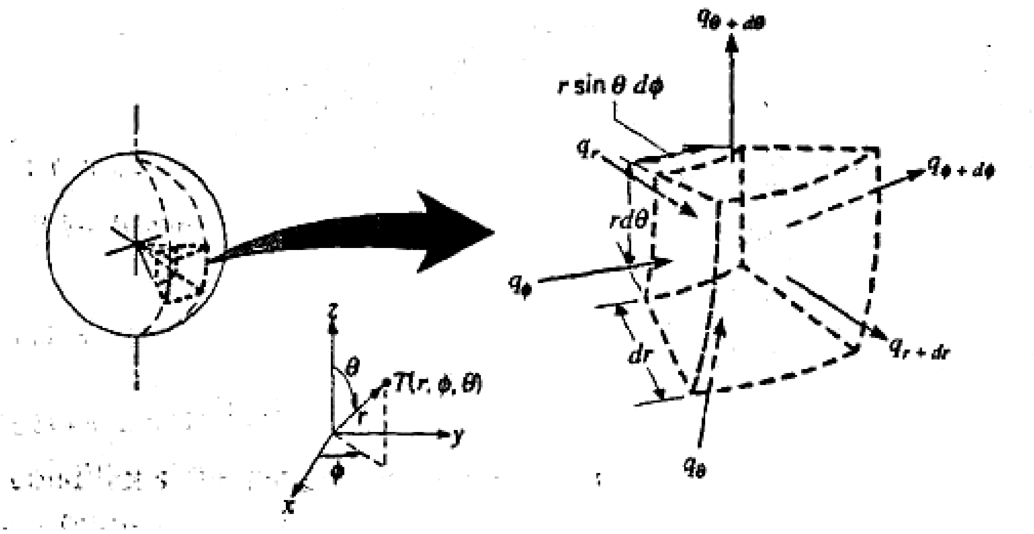


Figure 4.3 Differential control volume , dr.r sin θ dϕ .r dθ , for conduction analysis in spherical coordinates (r,ϕ,θ)

The above equations represent the general form of heat equations; these are Partial differential equation (PDE). As the heat equations are second order in the spatial coordinates, two boundary conditions must be expressed for each coordinate in order to describe the system. But since the equation is first order in time, only one condition, termed the initial condition, must be specified.

4.3 Special Cases of one Dimensional Conduction

In order to reach solutions for the conduction heat transfer equation in engineering applications, some assumptions may be made. These assumptions and approximations give reasonable results and accuracy for many engineering applications.

One of the mostly used approximations of the general heat equation is the one dimensional steady state heat transfer by conduction. As this section considers one dimensional heat flow therefore a single coordinate is needed to describe the temperature gradient and heat flow which are exclusively in that direction and if the temperature at any point is independent of time this system is to be considered one dimensional steady state heat transfer.

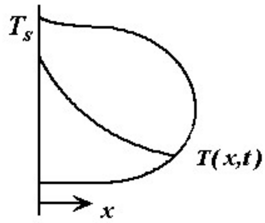
4.3.1 Boundary Conditions

Some of the boundary conditions usually met in heat transfer problems for one dimensional system are described below. These conditions are set at the surface x = 0, assuming transfer process in the positive direction of x- axis with temperature distribution which may be time dependent, designated as T(x, t).

Constant Surface Temperature

The surface is maintained at a fixed temperature T_s. It is commonly called a Dirichlet condition, which is the boundary condition of the first case. It can be approximated as a surface in contact with a solid or a liquid in a changing phase state (boiling, evaporating, melting or freezing) therefore the temperature is constant.

$$T(0,t) = T_s$$

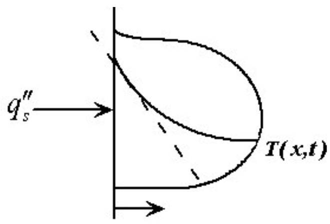


Constant Surface Heat Flux

In this case fixed or constant heat flux q" at the surface is described. At which the heat flux is a function of the temperature gradient at the surface by Fourier's law (Equation 4.1) that was stated before. This type of boundary condition is called Neumann condition. Examples of constant surface heat flux are described below.

a) Finite heat flux

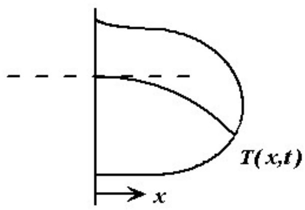
$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q_s'' \quad (4.14)$$



This type of boundary conditions may exist, for example, in the case of an electric heater attached to a surface.

b) Adiabatic heat flux

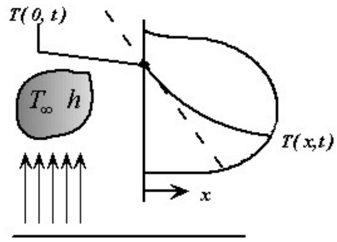
$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0_s \quad (4.15)$$



An example of the adiabatic heat flux is a surface which is perfectly insulated.

c) Convection surface condition

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0,t)] \qquad (4.16)$$



This represents the existence of convection heating (or cooling) at the surface and is determined by applying an energy balance on the surface by equating the amount of heat transferred by conduction with that transferred by convection which results yields Equation 4.16.

4.3.2 One Dimensional Steady state Conduction without Heat Generation

The assumptions made for this kind of analysis are:

- One dimensional
- Steady state
- No heat generation
- Constant material properties

Cartesian Coordinates

This model deals with a one dimensional steady state system with no heat generation. It may describe the heat flow through a plane wall. Therefore the temperature is a function of x coordinates and heat is transferred in that direction.

According to the pervious assumptions the general heat Equation 4.11 becomes

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0 \qquad (4.17)$$

By integrating this equation twice assuming constant thermal conductivity of the wall to determine the general equation

$$T(x) = C_1 x + C_2 \qquad (4.18)$$

In order to obtain the constants found in Equation 4.18 we have to introduce the appropriate boundary condition. From the analysis done above, we can choose the first case (the surface is maintained at a fixed temperature T_s)

$$T(0) = T_{s,1} \qquad T(L) = T_{s,2} \qquad (4.21)$$

Substituting these conditions in Equation 4.18 we obtain the two constants as follows
At x =0 we get

$$T_{s,1} = C_2$$

Similarly at x =L

$$T_{s,2} = C_1 L + C_2$$

$$T_{s,2} = C_1 L + T_{s,1}$$

We can put the constant C_1 in the following form

$$\frac{T_{s,2} - T_{s,1}}{L} = C_1 \tag{4.22}$$

Substituting in the general solution of the heat equation

$$T(x) = (T_{s,2} - T_{s,1})\frac{x}{L} + T_{s,1} \tag{4.23}$$

From Fourier's law the heat flux crossing the wall is expressed by

$$q_x'' = k\frac{dT}{dx}$$

Thus the amount of heat transferred through the wall of an area A is then obtained by

$$q_x = k\,A\frac{dT}{dx}$$

From the previous derivation the temperature gradient dT/dx could be obtained as

$$\frac{dT}{dx} = C_1 = \frac{T_{s,2} - T_{s,1}}{L} \tag{4.24}$$

$$q_x = k\,A(\frac{T_{s,2} - T_{s,1}}{L}) \tag{4.25}$$

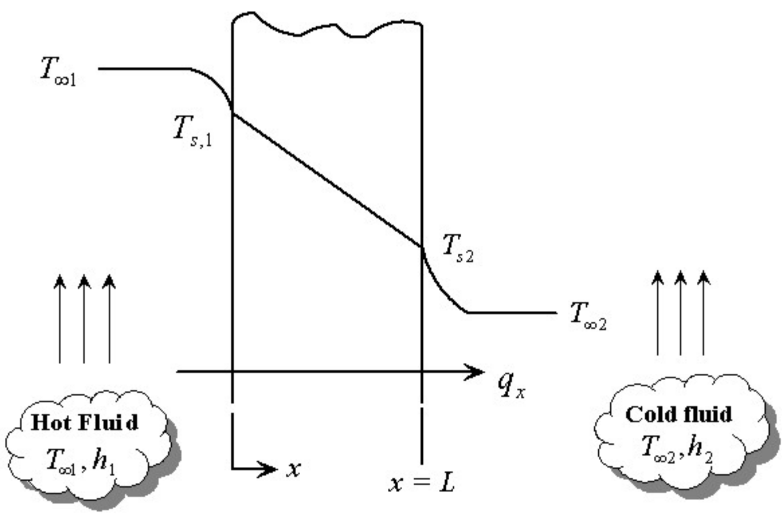
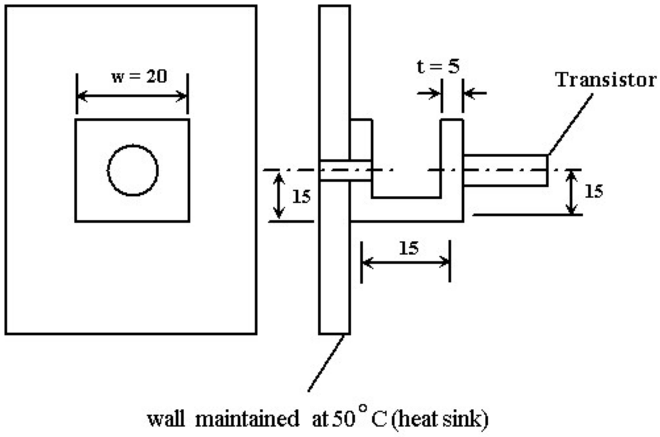


Figure 4.10 Heat transfer through a plane wall

Example 4.1: Calculate the maximum temperature the transistor base attains if it dissipates 7.5 W through the bracket shown in the Figure below. All dimensions are in mm and the bracket is made of duralumin.



Required:
The maximum temperature attained by the transistor base.

Solution:
Given
Dimension on the figure.
q = 7.5 W
t_w = 50 º C
This problem could be approximated to one dimensional steady state conduction.
k = 164 W/m.K
As the dimensions of the bracket is very small we can consider that the transfer is in one dimension through the sides of the bracket for the dimensions shown on the figure
L = 15 +15 + 15 = 45 mm = 0.045 m
w = 20 mm = 0.02 m
δ = 5 mm = 0.005 m
$A = w \times \delta = 1 \times 10^{-4}\ m^2$
From the above values the only unknown in equation 4.23 is the temperature of the transistor base.
$q = k A (t_b - t_w)/L$
$t_b = t_w + (qL/kA)$
$t_b = 50 + (7.5 \times 0.045/164 \times 10^{-4})$
t_b = 70.58 ºC

Note:

Usually the maximum allowable temperature is 100 ºC; therefore, the transistor temperature base is safe with a reasonable value which compensates the approximation done in the solution.

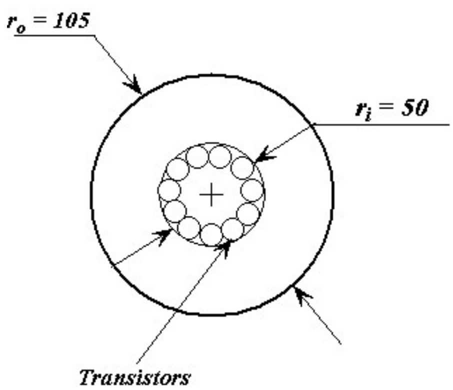
Cylindrical Coordinates
Considering the above assumptions, dT/dr is constant and heat flow only in one spatial coordinates the general form becomes

$$q = \frac{2\pi k L (T_1 - T_2)}{\ln(r_2 / r_1)} \qquad (4.26)$$

Where subscripts 1 and 2 refer to the inner and the outer surfaces respectively

Example 4.2: A hollow stainless (25 % Cr, 20 % Ni) steel cylinder 35 mm long has an inner diameter of 50 mm and outer diameter of 105 mm. a group of resistors that generate 10 W is to be mounted on the inside surface of the cylinder as shown in figure if the resistors temperature is not to exceed 100 ºC find the maximum allowed temperature on the outer surface of the cylinder.

Solution:
From appendix the thermal conductivity of 25 % Cr, 20 % Ni stainless steel is 12.8 W/m.K from Equation 4.26; the only unknown is the temperature of the outside surface of the cylinder.



$$T_o = T_i - \frac{q(\ln(r_o / r_i))}{2\pi kL}$$

T_o= 100 – 10 × (ln (52.5/25)/ (2π × 12.8 × 0.035)
T_o = 100 – 2.64 = 97.36 ºC

Spherical Coordinates
The same assumptions are applied to the spherical system giving the following solution

$$q_r = \frac{(T_i - T_o)}{\frac{(1/r_o - 1/r_i)}{4\pi k}}$$

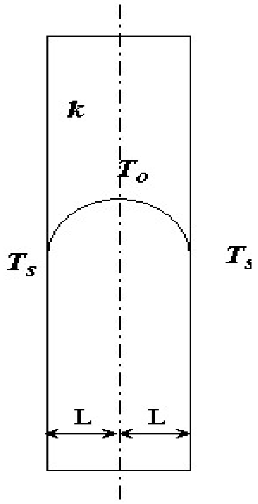
Where the subscripts i and o refer to the inside and the outside surfaces respectively.

4.3.3 One Dimensional Steady state Conduction with Uniform Heat Generation
The assumptions made for this kind of analysis are:
- One dimensional
- Steady state
- Uniform heat generation
- Constant material properties

Cartesian Coordinates

The heat equation becomes

$$k\frac{d^2T}{dx^2} + q''' = 0$$



Integrating and applying the boundary conditions described in figure we get

$$T = -\frac{q'''}{2k}x^2 + C_1 x + C_2$$

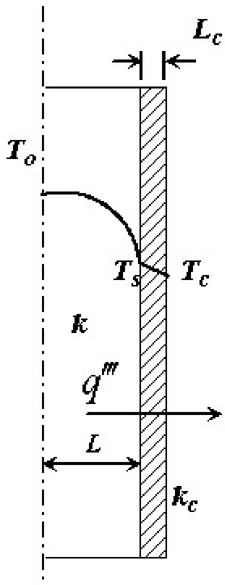
$C_1 = 0$ and $C_2 = T_s + (q''' L^2/2k)$

Then we get

$$T = -\frac{q'''}{2k}(L^2 - x^2) + T_s$$

The above equation differs for different wall construction and condition for an example if there is an outer cladding the temperature distribution through the wall will differ.

$q = q''' \times A \times L = T_s - T_c/(L_c/k_c A_c)$



Temperature distribution through the wall will be

$T = T_c + q''' L \times L_c / k_c + q''' / 2k(L^2 - x^2)$

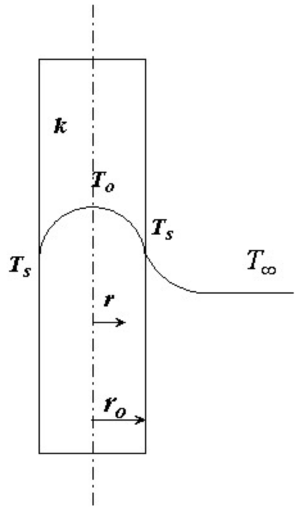
Similarly we can apply these cases on the other coordinate systems (cylindrical and spherical).

The boundary condition of each case affects the constants introduced in the equation of the temperature distribution leading to a change in the final formula.

Cylindrical Coordinates

The heat equation becomes

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k r \frac{\partial T}{\partial r}\right) + q''' = 0$$



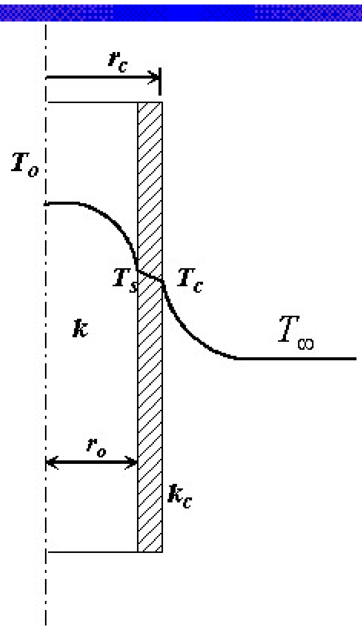
For the following boundary conditions
At $r = 0$ $dT/dr = 0$ where $C_1 = 0$
At $r = r_o$ $T = T_s$ where $C_2 = T_s + q''' r_o^2 / 4k$

We get the temperature distribution expression as follows
$T = T_s + q''' / 4k(r_o^2 - r^2)$

If the wall is subjected to a convective boundaries or it is cladded the same as in the plane wall the boundary conditions is to be applied and we are going to give the final form and the derivation is left to the student.

For a convective boundaries: $T = T_\infty + q''' / 4k (r_o^2 - r^2) + q''' r_o / 2h$

And if it is cladded: $T = T_\infty + \frac{q'''}{4k}(r_o^2 - r^2) + q'''\left[\frac{\ln(r_c / r_o)}{2k_c} + \frac{1}{2hr_c L}\right] r_o^2 \ L$



4.4 Extended Surfaces (Fins)

As clear from the rate equations that enhancing heat transfer could be done by several methods it could be by increasing the temperature difference or by increasing the heat transfer coefficient and also by increasing the surface area A, in this section we are going to deal with the last one, and this idea is done by adding a secondary surface to the primary surface and it is called extended surfaces, due to temperature gradient through the fins the heat transferred is decreased per unit area.

In electronic equipment cooling straight rectangular fins are mostly used and are done of good conducting material to attain the root temperature through the fins in order to increase the heat transferred.

Fins used in electronics cooling are usually used of aluminum and quite thin about 1.3 to 1.5 mm thick. Also in electronics cooling fins are usually considered of insulated tip. The heat transferred by fins are expressed in its effectiveness which is defined as

$\eta_f = q_f / q_{max}$

Where q_f is the heat actually transferred by the fin and q_{max} is the maximum heat could be dissipated by the fin and this happens when the fin has a uniform temperature equals to the root temperature t_r.

$\eta_f = (\tanh ml)/(ml)$

Where $m = \sqrt{2h(L+b)/k(b\ L)}$, l , L and b are defined on the fin sketch (Figure 4.11a).

Also the fin effectiveness could be considered as the fraction of the total surface area of the fin A_f that is effective for the heat transfer by convection maintained at root temperature t_r.

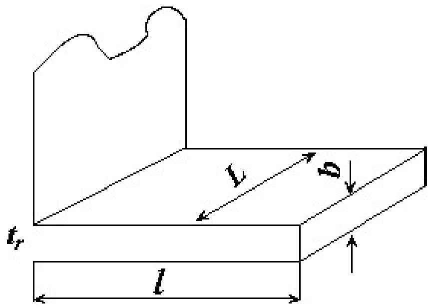
$\eta_f = A_{f,\,eff}/A_{f,tot}$

$q = h\,[(A_{tot} - A_f) + \eta_f A_f]\,(t_r - t_a)$

$= \eta_o A_{tot}\, h\,(t_r - t_a)$

Where $\eta_o = 1 - (A_f/A_{tot})\,(1 - \eta_f)$

If the fin is of convective tip a correction could be done as
$l_c = l + (b/2)$



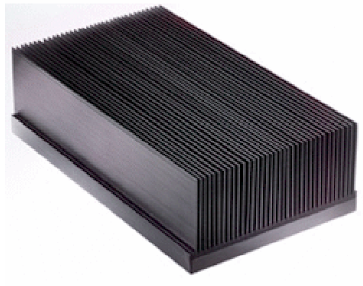


Figure 4.11a Straight rectangular fin dimensions	Figure 4.11b Typical heat sink with straight rectangular fins for electronic device

4.4.1 Fin Geometries

There are many types of plate-fin surfaces including plain fins, louvered fins, strip (or lanced offset) fins, wavy fins and pin fins. With 56 plate-fin surfaces tested, there is a wide range of fin geometries for which data is directly available. Figure 4.12 shows five common plate fin types and the critical fin dimensions.

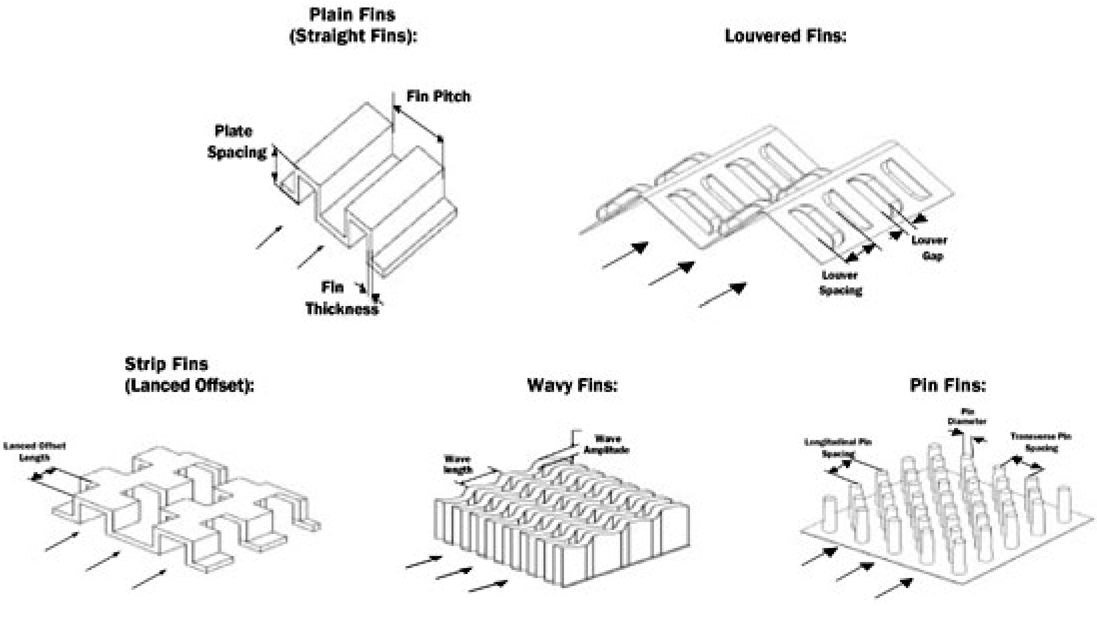


Figure 4.12 Fin descriptions

4.4.2 Factors Affected on the Fin Selection

With avionics power densities increasing, it becomes even more critical to design an optimum compact heat exchanger to remove the heat load most efficiently. There are many factors to consider in the selection of fin style and geometry. All of the parameters will discusses and should help the thermal engineer understand and trade these factors so that the best design may be obtained. There will always be some give and take among all these factors, but knowing and evaluating these factors is critical in obtaining an optimum design.

Using the Data

The data in Kays & London is presented as Fanning friction factor and Colburn factor vs Reynold's number. The Fanning friction factor also known as the skin friction coefficient, is defined as

$$f = \frac{\tau_o}{(1/2)\rho v^2} \tag{4.27}$$

It is related to the more common D'Arcy friction factor by

$$f_{D'Arcy} = 4f \tag{4.28}$$

The pressure drop, using the Fanning friction factor provided by Kays & London, is given by:

$$\Delta P = 4f\frac{L}{D_h}\left(\frac{\rho v^2}{2}\right) \tag{4.29}$$

The Colburn factor, j is defined by:

$$j = N_{St}\,N_{\mathrm{Pr}}^{2/3} = \frac{N_{Nu}}{N_{\mathrm{Re}}N_{\mathrm{Pr}}^{1/3}} \tag{4.30}$$

The film coefficient, using the Colburn factor provided by Kays & London is given by:

$$h = \frac{k}{D_h}\,jN_{\mathrm{Re}}N_{\mathrm{Pr}}^{1/3} \tag{4.31}$$

Design Practices

Now that you have all this good information on heat transfer and pressure drop for the different extended surfaces, you need to find the best way to use it. In the past, the thermal engineer would open up the Kays & London text book and look for a certain type of fin, such as pin, straight, lanced offset, or wavy, that worked for the combination of heat transfer, required pressure drop, and size constraints. This approach produces a design that works, but is hardly optimized for any of the considerations involved in designing the compact heat exchanger. A typical heat transfer plot of straight fins, from Kays and London, is shown in Figure 4.13.

Figure 4.13 allows the thermal engineer to calculate the heat transfer and pressure drop by picking off the j-Colburn factor and friction factor at the proper Reynolds number for a certain configuration, but it is unknown whether the proper design was chosen from a size and efficiency standpoint. The first thing the thermal engineer needs to do is prioritize the design factors. Pressure drop, heat transfer, size, weight and cost must be placed in order of importance. Hence, figures of merit were developed in an effort to compare the different fin configurations.

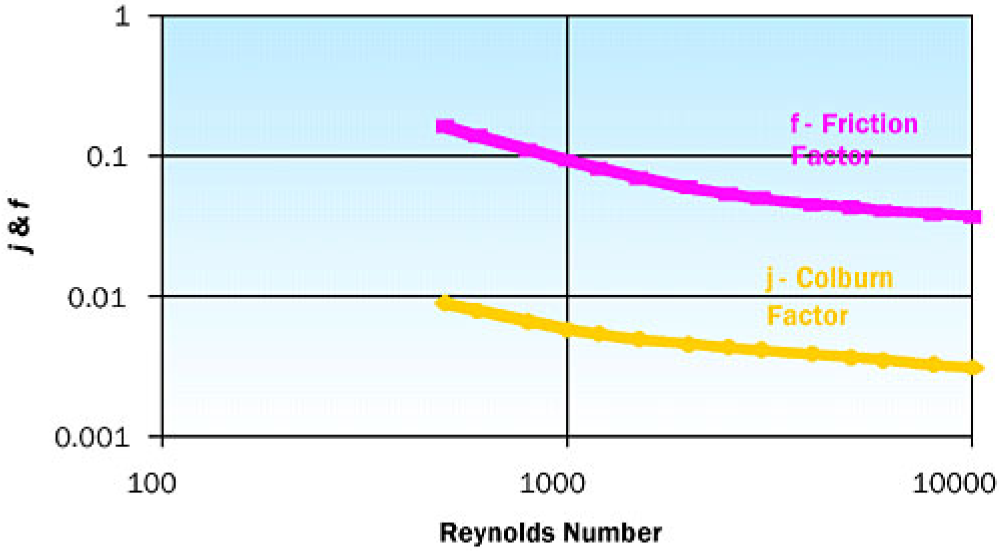


Figure 4.13 Finstock data # 14.77 from Kays & London (0.4 mm high, 0.15mm thick, 37.5 fins/mm, ST [0.330" high, 0.006" thick, 14.77 fpi, ST])

If pressure drop and heat transfer are the most critical factors in the compact heat exchanger design then a good figure of merit is heat transfer per unit of pressure drop, or in basic terms: j-Colburn factor divided by f-friction factor. Figure 4.14 shows a comparison of the different types of extended surfaces using heat transfer/pressure drop as the figure of merit.

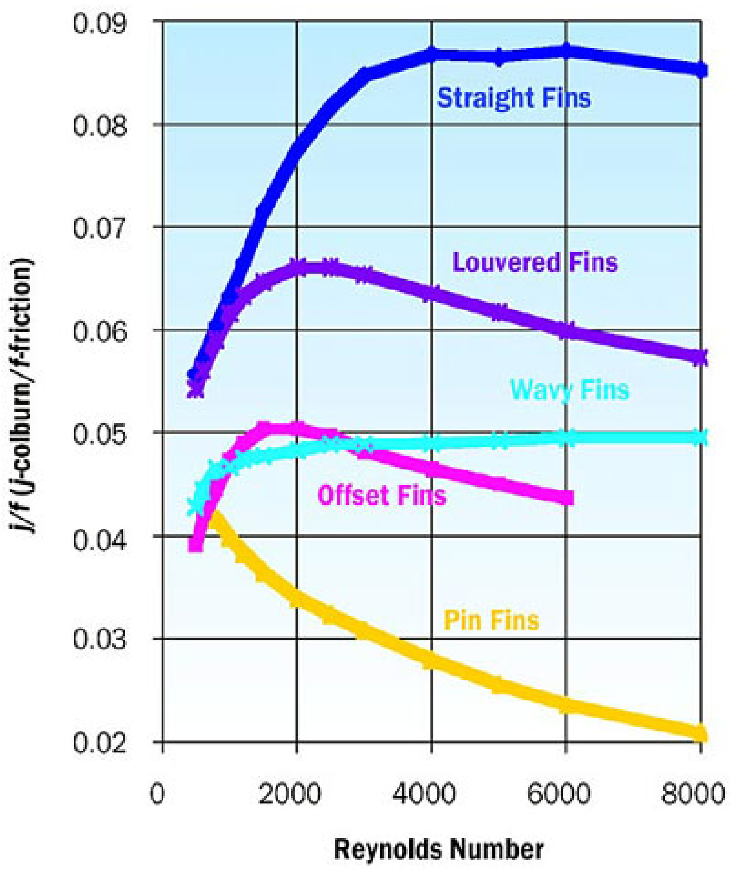


Figure 4.14 Heat transfer/pressure drop

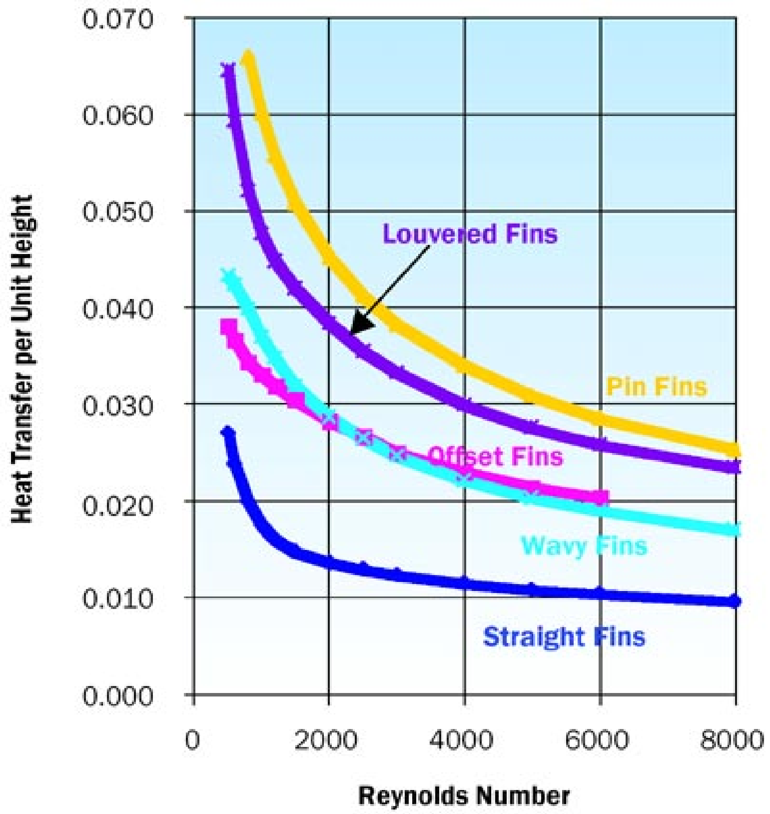


Figure 4.15 Size figure of merit

As can be seen from Figure 4.14, the efficiency of straight fins used in a heat exchanger is better than the other fin configurations over almost the whole range of typical Reynolds numbers. At a Reynolds number of 4000 the straight fin produces a three times better ratio of heat transfer per unit of pressure drop than the pin fin configuration. This figure of merit is based strictly on getting the best heat transfer for a given pressure drop.

Size is generally an important factor in the heat exchanger design. If size is the overall driving design factor then a good figure of merit is heat transfer per unit height. Figure 4.15 provides a comparison if size or height is the critical factor in the heat exchanger design. Figure 4.15 shows that, from a purely size standpoint, pin fins offer the smallest design for the best heat transfer while straight fins are the most inefficient from a heat transfer and smallest size constraint.

Weight is almost always an important factor in avionics cooling and designing a heat exchanger for an aircraft. A good figure of merit for weight is based on the heat transfer per weight unit. Figure 4.16 shows a comparison of fin configurations when weight is the critical factor in the design.

The last parameter that must be traded is cost, which is always an important factor in the design of a compact heat exchanger. It is also one of the more subjective areas. In general, pin fins, which are incorporated directly into the casting of the heat exchanger, are the least expensive. The other fin configurations, such as wavy, straight, offset louvered and cost about the same with some minor differences in set up costs.

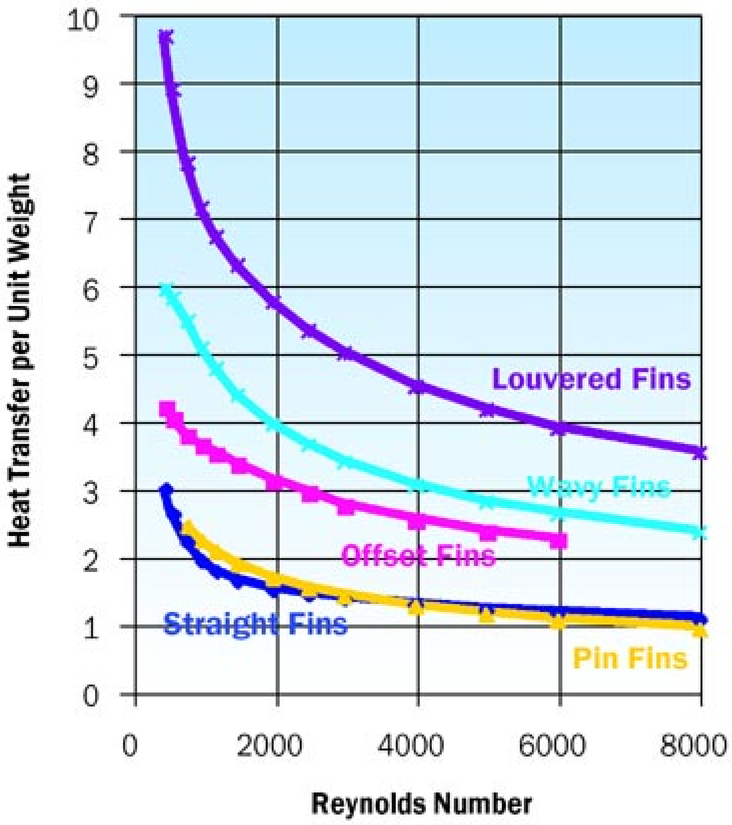


Figure 4.16 Weight figure of merit

A relative comparison of the fin configurations, based on all the factors discussed is critical in determining the proper design. All of the parameters are presented as individual design points, while the charts show the relative comparison for one parameter such as size, the assumption is made that pressure drop is unlimited as well as weight or cost. All of these parameters must be considered to obtain the proper design. The table above summarizes or ranks each factor so that the thermal engineer can get a relative view of all the parameters together. The rankings in Table 4.1 are from 1 to 5 with a ranking 1 being the most desirable and a ranking of 5 being the least desirable.

Table 4.1 Comparison of all parameters

Fin Configuration	ΔP	Size	Weight	Cost	Average
Straight	1	5	4	2	3
Offset	4	3	3	4	3.5
Pin	5	1	5	1	3
Wavy	3	4	2	3	3
Louver	2	2	1	5	2.5

With all of the parameters weighted equally the louvered fin configuration produces the best design for a compact heat exchanger. Another important factor is that even though the cost of the louvered fin is highest its cost is only slightly higher than the wavy, offset and straight fins. By contrast, the pin fin structure is an all or nothing configuration, with the highest figure of merits for cost and size, while performing extremely poorly in pressure drop and weight. So if cost and size are the number one priority without concern for pressure drop then the pin fins performs very well. Straight fins are also your best choice if pressure drop is the limiting factor in the design, which is usually a driving factor in avionics heat exchanger design. With all the parameters considered, all five fin configurations came out surprisingly equal, so a weighted average would be most appropriate in finding the proper design for a given set of prioritized factors.