11. Advanced Radiation

11.1 Gray Surfaces

The gray surface is a medium whose monochromatic emissivity (ε_λ) does not vary with wavelength. The monochromatic emissivity is defined as the ratio of the monochromatic emissive power of the body to monochromatic emissive power of a black body at same wave length and temperature.

$$\varepsilon_\lambda = E_\lambda / E_{\lambda,b} \quad (11.1)$$

But

$$E = \int_0^\infty \varepsilon_\lambda E_{\lambda,b} d\lambda$$

And

$$E_b = \int_0^\infty E_{\lambda,b} d\lambda = \sigma T^4$$

So that

$$\varepsilon = \frac{\int_0^\infty \varepsilon_\lambda E_{\lambda,b} d\lambda}{\sigma T^4} \quad (11.2)$$

Where: $E_{\lambda,b}$ is the emissive power of a black body per unit wave length.

If the gray body condition is imposed, that is, ε_λ = constant, so that:

$$\varepsilon = \varepsilon_\lambda = \text{constant} \quad (11.3)$$

11.2 Radiation Exchange between Gray Surfaces (ε = Constant)

The calculation of the radiation heat transfer between black bodies is relatively easy because all the radiant energy which strikes a surface is absorbed (reflectivity = 0). For non black bodies (such as gray bodies) the situation is much more complex, because all the radiant energy which strikes a surface will not be absorbed, part will be reflected back to another surface, and part will be reflected out of the system entirely. Thus, we need to define two new terms:

G = irradiation or total incident radiation (W/m^2)

J = radiosity or total radiation leaving the surface (W/m^2)

Our assumptions are that all surfaces considered in our analysis are diffuse, isothermal and that the radiations properties are constant over all surfaces as well as the irradiation and radiosity. Now the objective is to determine the net radiation heat transfer (q) from each surface.

From Figure 11.1 (a) and the definition of the radiosity we can mathematically define the radiosity as:

$$J = \varepsilon E_b + \rho G \quad (11.4)$$

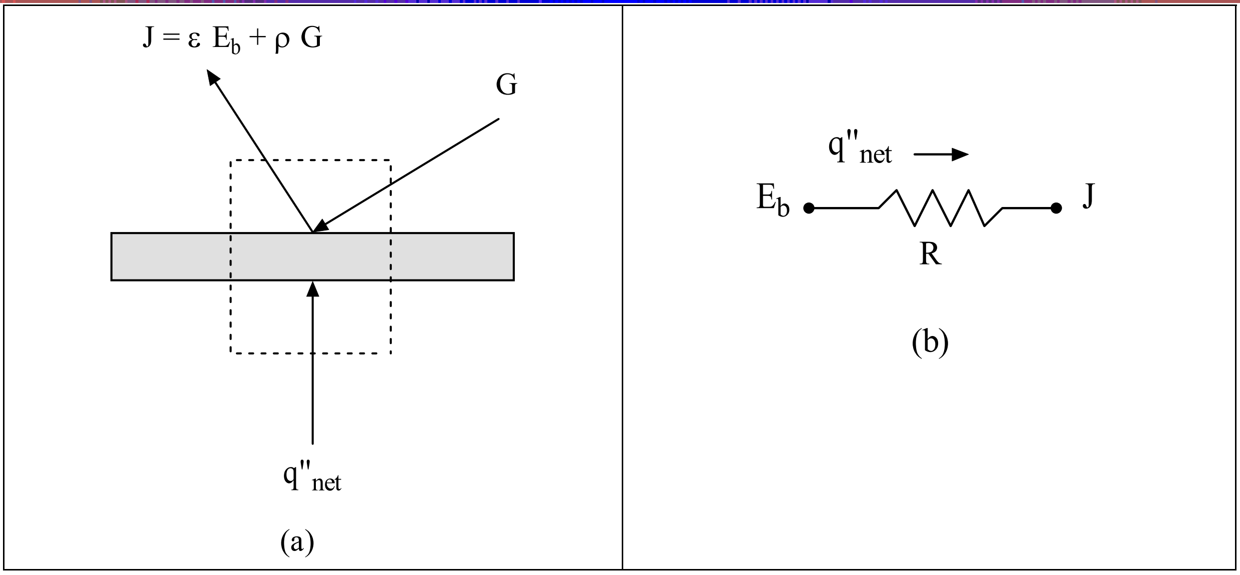


Figure 11.1(a) surface energy balance, (b) surface resistance in the radiation network method

Since the transimissivity is assumed to be zero then the reflectivity may be expressed as

$$\rho = 1 - \alpha$$

Knowing that for a gray surface $\alpha = \varepsilon$, then

$$\rho = 1 - \varepsilon$$

An enhanced mathematical formula for the radiosity will be:

$$J = \varepsilon E_b + (1-\varepsilon) G$$

Or,

$$G = \frac{J - \varepsilon E_b}{(1-\varepsilon)}$$

From the energy balance shown in Figure11.1 (a) the net radiation flux leaving the surface is

$$q''_{net} = J - G = J - \frac{J - \varepsilon E_b}{(1-\varepsilon)} = \frac{\varepsilon (E_b - J)}{(1-\varepsilon)} \tag{11.5}$$

The general equation for net radiation heat transfer leaving the surface is

$$q_{net} = \frac{(E_b - J)}{\left(\frac{1-\varepsilon}{\varepsilon A}\right)} = \frac{(E_b - J)}{R} = \frac{\text{Potential difference}}{\text{Surface resistance}} \tag{11.6}$$

The analogy between radiation heat transfer and electric circuit is shown in Figure 11.1 (b). The heat flows as the current through a resistance.

Now consider the exchange of radiant energy between two surfaces 1, 2 as shown in Figure 11.2.

The total radiation which leaves A_1, reaching A_2 is

$$J_1 A_1 F_{12}$$

Similarly the total radiation which leaves A_2, reaching A_1 is

$$J_2 A_2 F_{21}$$

The net radiation between the two surfaces is

$$q_{1-2} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$$

Recalling the reciprocity relation

$$A_1 F_{12} = A_2 F_{21}$$

Then,

$$q_{1-2} = (J_1 - J_2) A_1 F_{12}$$

$$q_{1-2} = q_{net} = \frac{J_1 - J_2}{\left(\frac{1}{A_1 F_{12}}\right)} = \frac{J_1 - J_2}{R} = \frac{\text{Potential difference}}{\text{Space resistance}} \qquad (11.7)$$

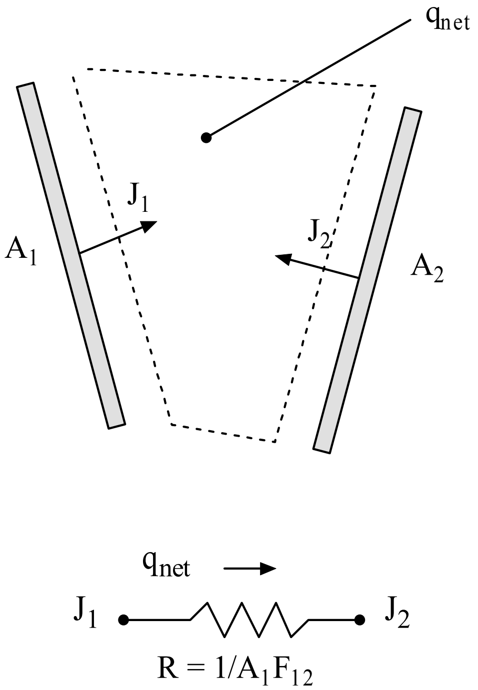


Figure 11.2 Radiation exchange between two surfaces

Now we can write the general equation which connect between the surface and space resistance between two surfaces as

$$q_{1-2} = \frac{E_{b1} - E_{b2}}{\left(\frac{1-\varepsilon_1}{\varepsilon_1 A_1}\right) + \left(\frac{1}{A_1 F_{12}}\right) + \left(\frac{1-\varepsilon_2}{\varepsilon_2 A_2}\right)} = \frac{\sigma (T_1^4 - T_2^4)}{\left(\frac{1-\varepsilon_1}{\varepsilon_1 A_1}\right) + \left(\frac{1}{A_1 F_{12}}\right) + \left(\frac{1-\varepsilon_2}{\varepsilon_2 A_2}\right)} \qquad (11.8)$$

The resistances of Equation 11.8 are shown in Figure 11.3 below.

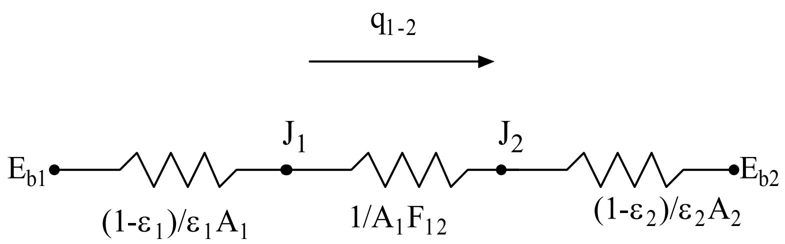
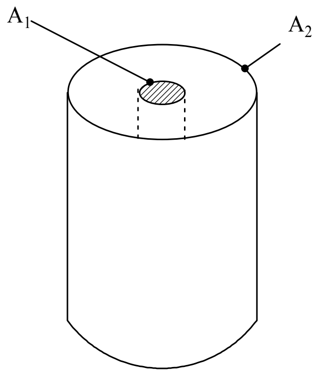


Figure 11.3 Radiation network for two surfaces

11.2.1 Special Cases for Two Gray Surfaces

(a) Long (infinite) concentric cylinders:



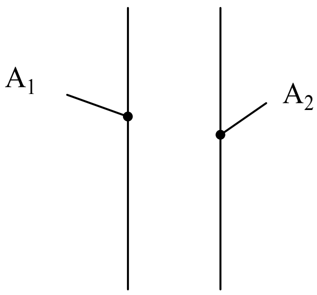
By applying summation rule

$$F_{11} + F_{12} = 1$$

$$F_{11} = 0 \quad \text{so that} \quad F_{12} = 1$$

$$q_{1-2} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + (\frac{1}{\varepsilon_2} - 1)\frac{A_1}{A_2}}$$

(b) Two long (infinite) parallel planes:



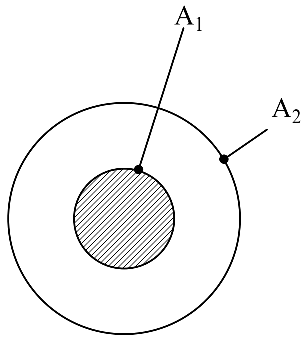
By applying summation rule

$$F_{11} + F_{12} = 1$$

$$F_{11} = 0 \quad \text{so that} \quad F_{12} = 1 \text{ and } A_1 = A_2$$

$$q_{1-2} = \frac{\sigma A \; (T_1^4 - T_2^4)}{(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1)}$$

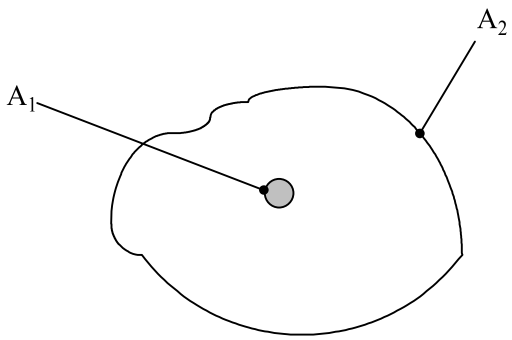
(c) Two concentric spheres:



By applying summation rule to gives $F_{12} = 1$ and knowing that $A_1/A_2 = (r_1/r_2)^2$

$$q_{1-2} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + (\frac{1}{\varepsilon_2} - 1)\left(\frac{r_1}{r_2}\right)^2}$$

(d) Small body in large enclosure:



$A_1/A_2 \approx 0$

By applying summation rule to gives $F_{12} = 1$

$$q_{1-2} = \sigma \varepsilon_1 A_1 (T_1^4 - T_2^4)$$

11.3 Radiation Exchange between Three Gray Surfaces

If we have three surfaces the network for this system is shown in Figure 11.4.

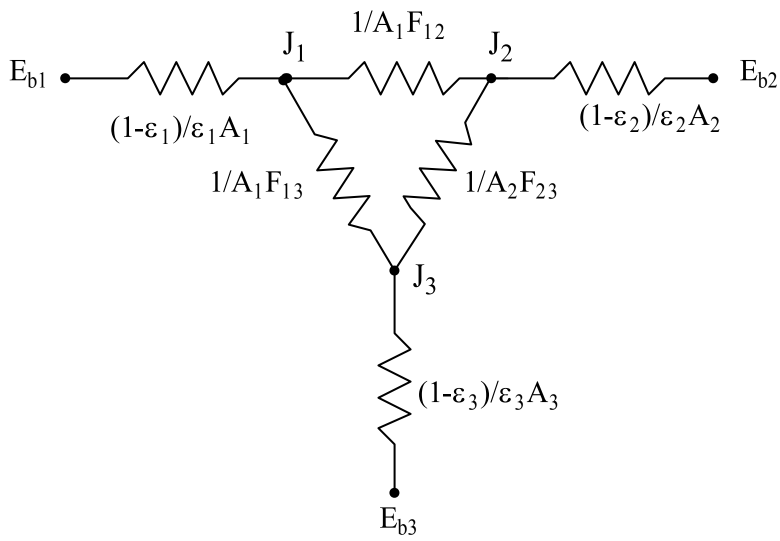


Figure 11.4 Network for three surfaces

By applying the Kirchhoff's law: "Summation of currents entering each node equals to zero"

At node J_1:

$$\frac{E_{b1}-J_1}{\left(\frac{1-\varepsilon_1}{\varepsilon_1 A_1}\right)}+\frac{J_2-J_1}{\left(\frac{1}{A_1F_{12}}\right)}+\frac{J_3-J_1}{\left(\frac{1}{A_1F_{13}}\right)}=0$$

At node J_2:

$$\frac{E_{b2}-J_2}{\left(\frac{1-\varepsilon_2}{\varepsilon_2 A_2}\right)}+\frac{J_3-J_2}{\left(\frac{1}{A_2F_{23}}\right)}+\frac{J_1-J_2}{\left(\frac{1}{A_1F_{12}}\right)}=0$$

At node J_3:

$$\frac{E_{b3}-J_3}{\left(\frac{1-\varepsilon_3}{\varepsilon_3 A_3}\right)}+\frac{J_2-J_3}{\left(\frac{1}{A_2F_{23}}\right)}+\frac{J_1-J_3}{\left(\frac{1}{A_1F_{13}}\right)}=0$$

Solving the above three equations for the radiosity yield the values of them, thus we could calculate the energy exchange between different surfaces.

11.3.1 Special Cases for Three Gray Surfaces

(a) Two surfaces inside very large enclosure:

Due to a very large area ($A \longrightarrow \infty$) the surface resistance approaches zero, which makes it behave like a black body with $\varepsilon = 1$, and will have $J = E_b$ because of the zero surface resistance.

An example network for this case may be as shown in Figure 11.5.

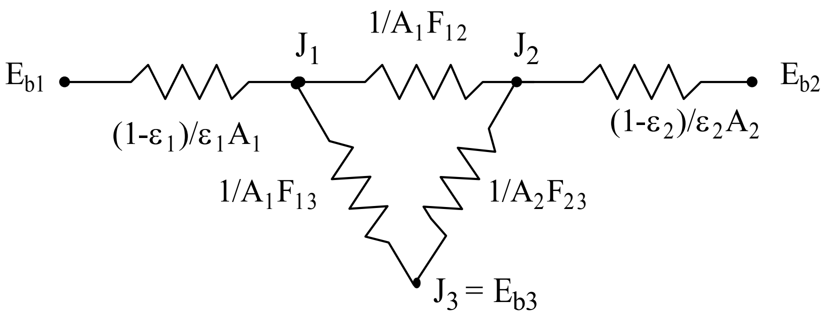


Figure 11.5 Network for surfaces inside enclosure

(b) Insulated surfaces:

If a surface is perfectly insulated, or re-radiates the entire energy incident upon it, it has zero heat flow and the potential across the surface resistance is zero surface resistance. In effect, the J node in the network is floating, i.e., it does not draw any current. An example network for this case may be as shown in Figure 11.6(a) and may be simplified to Figure 11.6(b).

The total circuit resistance of the circuit is

$$R_{total} = R_1 + \frac{1}{\left(\frac{1}{R_2}\right) + \left(\frac{1}{R_3 + R_4}\right)} + R_5$$

So that the net radiation heat exchange is

$$q_{net} = \frac{E_{b1} - E_{b2}}{R_{total}} = \frac{\sigma(T_1^4 - T_2^4)}{R_{total}} \qquad (11.9)$$

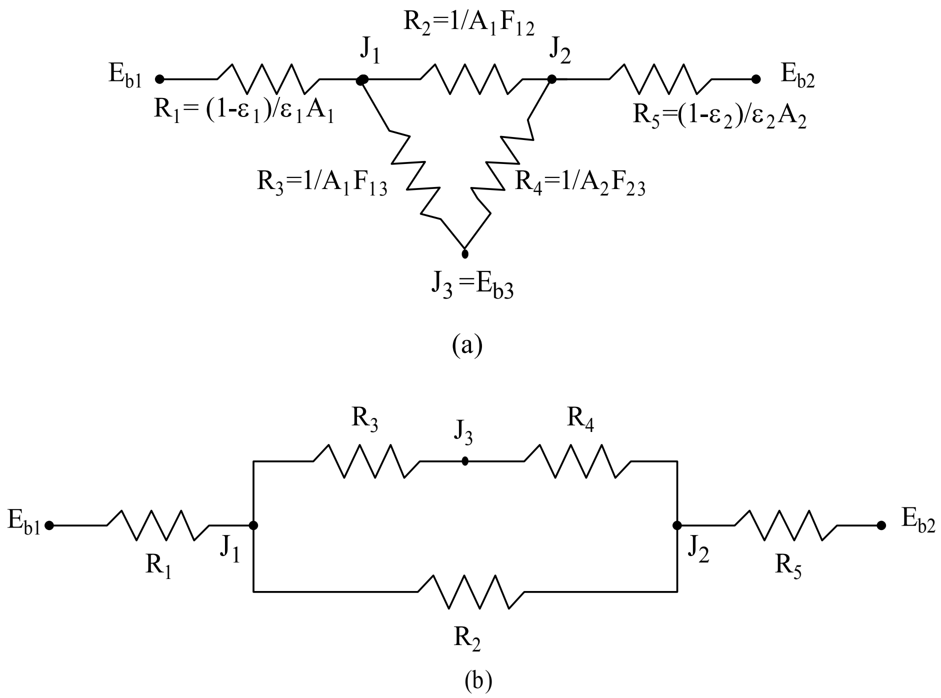


Figure 11.6 (a) Network for two surfaces with another insulated surface.
(b) Simplification for this network.

11.4 Radiation Shields

If it is desired to minimize radiation heat transfer between two surfaces, it is a common practice to place one or more radiation shields between them as shown in Figure 11.7. These shields do not deliver or remove any heat from the overall system; they only place another resistance in the heat flow path so that the overall heat transfer is retarded.

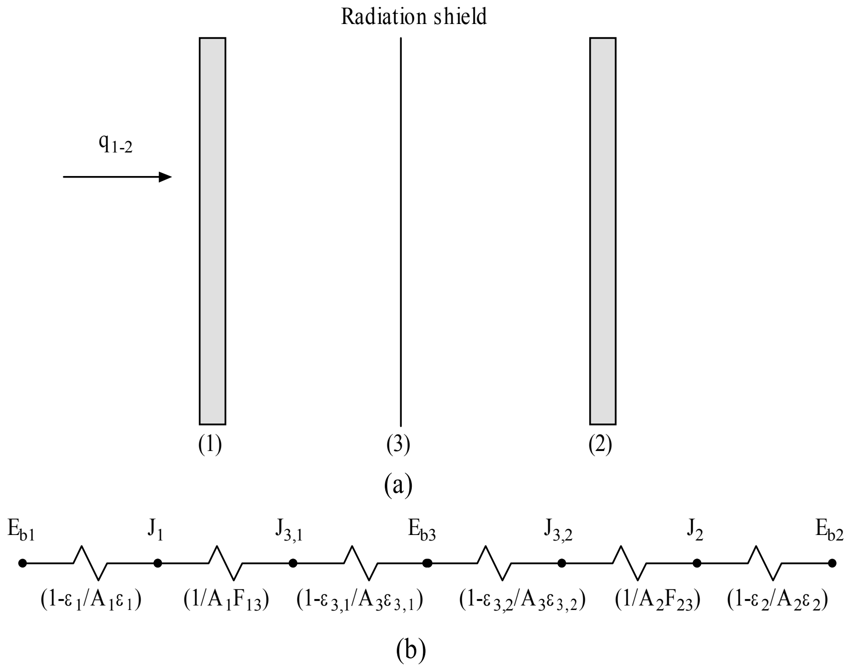


Figure 11.7 (a) Radiation between parallel planes with radiation shield,
(b) Network representation.

The general equation for the net radiation heat transfer is

$$q_{1-2} = \frac{E_{b1} - E_{b2}}{\left(\frac{1-\varepsilon_1}{A_1\varepsilon_1}\right) + \left(\frac{1}{A_1F_{13}}\right) + \left(\frac{1-\varepsilon_{3,1}}{A_3\varepsilon_{3,1}}\right) + \left(\frac{1-\varepsilon_{3,2}}{A_3\varepsilon_{3,2}}\right) + \left(\frac{1}{A_2F_{23}}\right) + \left(\frac{1-\varepsilon_2}{A_2\varepsilon_2}\right)}$$

$$= \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1-\varepsilon_1}{A_1\varepsilon_1}\right) + \left(\frac{1}{A_1F_{13}}\right) + \left(\frac{1-\varepsilon_{3,1}}{A_3\varepsilon_{3,1}}\right) + \left(\frac{1-\varepsilon_{3,2}}{A_3\varepsilon_{3,2}}\right) + \left(\frac{1}{A_2F_{23}}\right) + \left(\frac{1-\varepsilon_2}{A_2\varepsilon_2}\right)} \quad (11.10)$$

If surfaces are large (Infinite) and close together, so that $A_1 \approx A_2 \approx A_3 = A$, and $F_{13} = F_{23} = 1$
Then the net radiation heat transfer is.

$$q_{1-2} = \frac{\sigma A(T_1^4 - T_2^4)}{2 + \left(\frac{1-\varepsilon_1}{\varepsilon_1}\right) + \left(\frac{1-\varepsilon_{3,1}}{\varepsilon_{3,1}}\right) + \left(\frac{1-\varepsilon_{3,2}}{\varepsilon_{3,2}}\right) + \left(\frac{1-\varepsilon_2}{\varepsilon_2}\right)} \quad (11.11)$$

Example 11.1: A PCB maintained at 45 ºC is exposed to a parallel cold plate which is maintained at 10 ºC. Each plate is 0.2 × 0.15 m, the plates are placed 0.04 m apart. Consider the PCB and Cold plate as black bodies. What is the net radiant heat exchange between the PCB and the Cold plate?

Solution:

From the Figure A.2 with the ratios

X = 0.2 / 0.04 = 5 and Y = 0.15 / 0.04 = 3.75

We can read $F_{12} = 0.64$

Using Equation 10.17 the net radiant heat transfer is

$$\begin{aligned} q_{net} &= \sigma A_1 F_{12}(T_1^4 - T_2^4) \\ &= (5.67x10^{-8})(0.2x0.15)(0.64)(318^4 - 283^4) \\ &= 4.15 \text{ W} \end{aligned}$$

Example 11.2: The vertical side of an electronics box is 40 × 30 cm with the 40 cm side vertical. What is the maximum heat transfer that could be dissipated by this side if its temperature is not to exceed 60 ºC in an environment and surrounding of 40 ºC, if its emissivity is 0.8?

Solution:

The total heat transfer is due to natural convection and radiation heat transfer so that

$$q_{tot} = q_{conv.} + q_{rad.}$$

(a) Convection heat transfer calculation:

$$q_{conv.} = \bar{h}A(T_S - T_\infty)$$

Evaluate the properties of the air at the film temperature (T_f = 100/2 = 50 °C)
ν = 18.2 x 10^{-6} m^2/s.
k = 0.028 W/m. °C
Pr= 0.7038
C_p= 1008 J/kg. °C
β = 1/323 = 3.096 x10^{-3}
The characteristic length (L) = 0.4 m

The Gr Pr product is

$$Gr\,\mathrm{Pr} = \frac{9.81 \times 3.096 \times 10^{-3} \times 20(0.4)^3}{(18.2 \times 10^{-6})^2} 0.7038 = 82.6 \times 10^6$$

From the Nusselt number

$$\overline{Nu} = \frac{\bar{h}L}{k} = c\left(Gr\,\mathrm{Pr}\right)^m$$

With the constants

$$\mathrm{c} = 0.59 \quad \text{and} \quad \mathrm{m} = 0.25$$

Then;

$$\overline{Nu} = 0.59\left(82.6 \times 10^6\right)^{0.25}$$
$$= 56.25$$

So that the average heat transfer coefficient is

$$\bar{h} = \frac{\overline{Nu}\,k}{L} = \frac{56.25 \times 0.028}{0.4} = 3.94\ \mathrm{W/m^2.^oC}$$

The convection heat transfer is

$$q_{conv.} = 3.94 \times (0.3 \times 0.4)(60 - 40) = 9.456\ \mathrm{W}$$

(b) Radiation heat transfer calculation: (Small body in large enclosure)

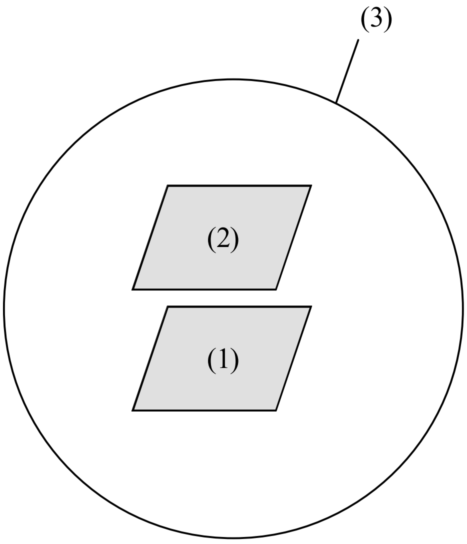
$$q_{rad.} = \sigma\varepsilon\ A\ (T_S^4 - T_{surrounding}^4)$$
$$= 5.67 \times 10^{-8}(0.8)(0.3 \times 0.4)(333^4 - 313^4)$$
$$= 14.688\ \mathrm{W}$$

The maximum heat transfer that could be dissipated by this side is

$$q_{tot.} = 9.456 + 14.688 = 24.144\ \mathrm{W}$$

Example 11.3: Two parallel PCBs 0.2 × 0.2 are spaced 0.1 m apart. One PCB is maintained at 55 ºC and the other at 40 ºC. The emissivities of the PCBs are 0.2 and 0.5, respectively. The PCBs are located inside a large chassis, the inside walls of chassis are maintained at 30 ºC. The PCBs exchange heat with each other and with the chassis, but only the PCBs surfaces facing each other are to be considered in the analysis. Calculate the net transfer to PCBs and to the chassis.

Schematic:



Solution:

This case resembles two surfaces inside a large enclosure case.

$T_1 = 328$ K $\quad A_1=0.04$ m^2 $\quad \varepsilon_1= 0.2$
$T_2 = 313$ K $\quad A_1=A_2=0.04$ m^2 $\quad \varepsilon_1= 0.5$
$T_3 = 303$ K

From Figure A.2 with the ratios

X = 0.2/0.1 =2 and Y = 0.2/0.1 = 2

So that $F_{12} = F_{21} = 0.42$

$F_{13} =1- F_{12} = 0.58$
$F_{23} =1- F_{21} = 0.58$

The network for this case is typically as that of Figure 11.5, and then the resistances in the network are:

$$\frac{1-\varepsilon_1}{\varepsilon_1 A_1} = \frac{1-0.2}{(0.2)(0.04)} = 100$$

$$\frac{1-\varepsilon_2}{\varepsilon_2 A_2} = \frac{1-0.5}{(0.5)(0.04)} = 25$$

$$\frac{1}{A_1 F_{12}} = \frac{1}{(0.04)(0.42)} = 59.5$$

$$\frac{1}{A_1 F_{13}} = \frac{1}{A_2 F_{23}} = \frac{1}{(0.04)(0.58)} = 43.1$$

At node J_1: $$\frac{E_{b1}-J_1}{100}+\frac{J_2-J_1}{59.5}+\frac{E_{b3}-J_1}{43.1}=0 \quad (1)$$

At node J_2: $$\frac{E_{b2}-J_2}{25}+\frac{E_{b3}-J_2}{43.1}+\frac{J_1-J_2}{59.5}=0 \quad (2)$$

But,

$E_{b1}=\sigma T_1^4=656.26\ \text{W/m}^2$

$E_{b2}=\sigma T_2^4=544.2\ \text{W/m}^2$

$E_{b3}=J_3=\sigma T_3^4=477.92\ \text{W/m}^2$

Now both Equations (1) and (2) have two unknowns J_1 and J_2 which may be solved simultaneously to give

$J_1 = 528.27\ \text{W/m}^2$
$J_2 = 521.63\,\text{W/m}^2$

The total heat lost by PCB number 1 is

$$q_1=\frac{E_{b1}-J_1}{\left(\dfrac{1-\varepsilon_1}{\varepsilon_1 A_1}\right)}=\frac{656.26-528.27}{100}=1.28\,\text{W}$$

The total heat lost by PCB number 2 is

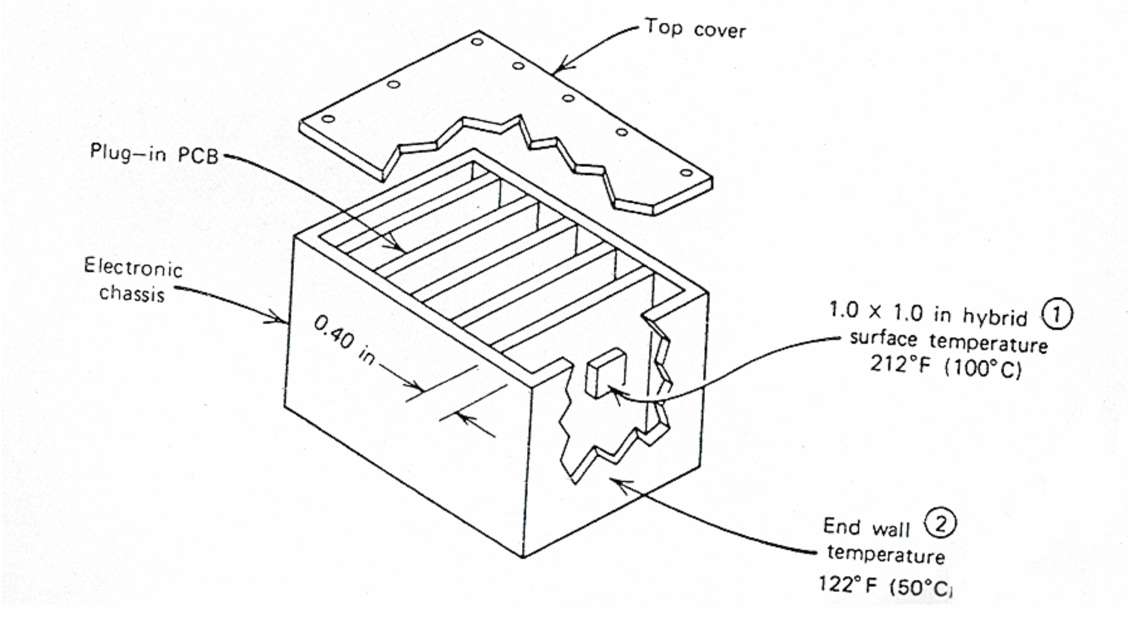
$$q_2=\frac{E_{b2}-J_2}{\left(\dfrac{1-\varepsilon_2}{\varepsilon_2 A_2}\right)}=\frac{544.2-521.63}{25}=0.903\,\text{W}$$

From an overall-balance we must have

$$q_3=q_1+q_2=2.183\,\text{W}$$

Example 11.4: A square flat pack hybrid 2.54 cm side, 0.457cm high have a maximum allowable temperature of 100 ºC mounted on the end of a PCB so that it faces the end wall of an electronic chassis, which has a temperature of 50 ºC, as shown in the following figure. The hybrid is about 1.02 cm from the wall, so that natural convection and conduction heat transfer will be negligible. Determine the maximum allowable power dissipation for the hybrid with ($\varepsilon = 0.8$) and without ($\varepsilon = 0.066$) a conformal coating.

Schematic:



Solution:

The hybrid flat pack may be assumed as a small body in large enclosure case.

(a) The heat transfer from the hybrid without coating is:

$$q = \sigma \varepsilon_1 A_1 (T_1^4 - T_2^4)$$

$$q = 5.67 \times 10^{-8} \times 0.066 \times (0.0254)^2 \left(373^4 - 323^4\right) = 0.02 \text{ W}$$

(b) The heat transfer from the hybrid with coating.

$$q_{coat} = \frac{0.8}{0.066} q_{without\ coat} = \frac{0.8}{0.066} \times 0.02 = 0.242 \text{ W}$$

Comment:
Adding a conformal coating to the hybrid will increase its heat transfer capability, thus allowing for extra heat dissipation. This might be regarded as a plus as it allows extra packing of electronic components inside this hybrid.