10. Radiation Heat Transfer

10.1 Introduction

Radiation heat transfer plays a major role in the cooling of electronics. As well as conduction and convection heat transfer, radiation is an equally important mode.
Also the thermal radiation has many applications such as engine cooling, furnaces, boilers, piping and solar radiation.
The thermal radiation transferred by electromagnetic waves, called photons, is emitted by bodies due to temperature differences. All surfaces continuously emit and absorb radiative energy by lowering or raising their molecular energy levels.
The electromagnetic waves travel through any medium or in vacuum at the speed of light (c), which equals 3×10^8 m/s. this speed equal to the product of the wave length λ and frequency ν

$$c = \lambda \nu$$

The metric unit for the wave length λ is centimeters or angstroms (1 angstrom = 10^{-8} centimeter). A portion of the electromagnetic spectrum is shown in Figure 10.1.

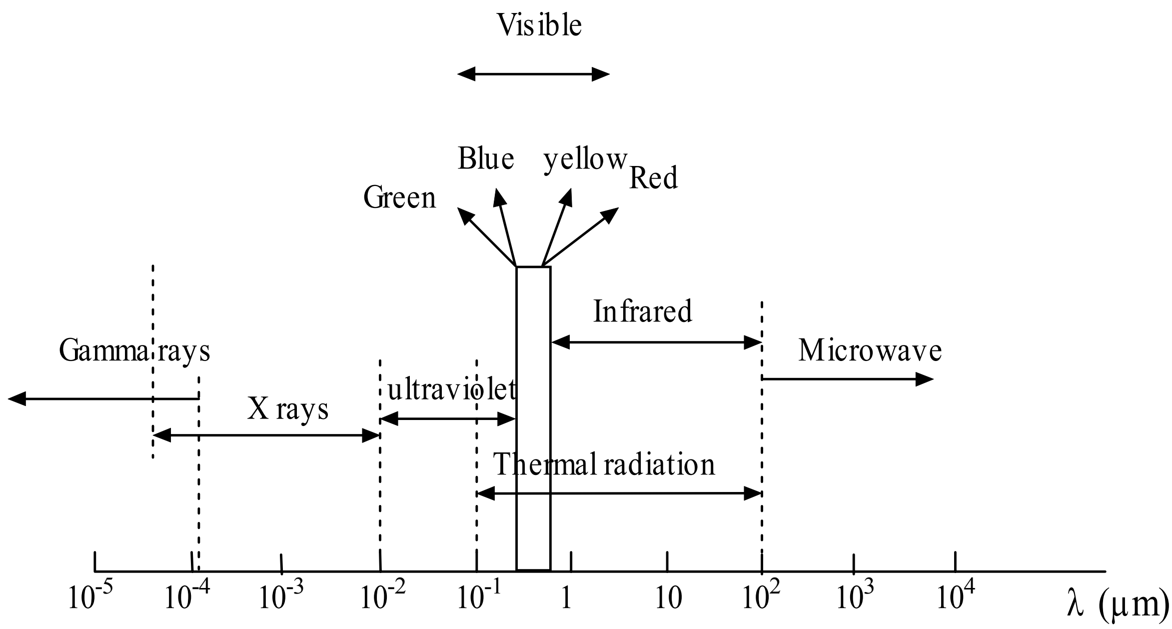


Figure 10.1 Electromagnetic spectrum

10.2 Blackbody Radiation

Materials used in electronic hard ware may be classified as black or gray surfaces.
The ideal surface is known as a “blackbody” or “black surface,” since it absorbs or emits 100% of the all incoming radiation.
A rough surface has higher emittance than the same surface when it is smooth. Also, a finned surface has a higher emittance than does the same surface without fins. In both cases the emittance increases because the surfaces have many small cavities. These cavities act as many small partial black bodies.

For example a small hole in a hollow sphere is often used to represent a black body. The energy enters the small opening and strikes the opposite wall, where part of the energy is absorbed and part is reflected. The reflected energy again strikes the opposite wall, where part of the energy is absorbed and part is

reflected. This process continues until all of the energy is absorbed, as shown in Figure10.2.
The total amount of radiative energy emitted from a surface into all directions above it is termed emissive power; we distinguish between spectral (at a given wavelength □, per unit wavelength and per unit surface area) and total (encompassing all wavelengths) emissive power. The magnitude of emissive power depends on wavelength □, temperature T, and a surface property, called emissivity □, which relates the ability of a surface to emit radiative energy to that of an ideal surface, which emits the maximum possible energy (at a given wavelength and temperature).
The spectral distribution of the emissive power $E_{\lambda,b}$ of a black surface is given by Planck's law is of the form

$$E_{\lambda,b} = \frac{C_1}{\lambda^5[e^{(C_2/\lambda T)} - 1]} \qquad (10.1)$$

Where the radiation constants or some times called Planck function constants are C_1= 3.742 x10^8 W.µm^4/m^2 and C_2=1.439 x10^4 µm.k.

The total emissive power E_b of a blackbody is given by Stefan-Boltzmann law. Expressed as

$$E_b = \int_0^\infty E_{b,\lambda} d\lambda$$

Integration yields;

$$E_b = \sigma T^4 \qquad (10.2)$$

Where the temperatures in the Kelvin's and the Stefan-Boltzmann constant (σ) has the numerical value

$$\sigma = 5.67 \text{ x } 10^{-8} \text{ W/m}^2\text{.K}$$

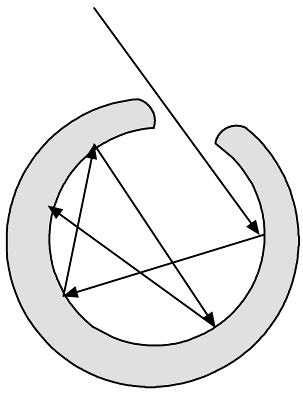


Figure 10.2 Constructing a black body enclosure

Figure 10.3 shows the spectral emissive power $E_{\lambda,b}$ as a function of temperature and wavelength, the maximum $E_{\lambda,b}$ is corresponding to wave length λ_{max} which depends on surface temperature.
The nature of this dependence may be obtained by differentiating the Equation 10.1 with respect to λ and setting the result equal to zero, we obtain

$$\lambda_{max} T = 2897.6 \text{ µm.K} \qquad (10.3)$$

The Equation 10.3 is known as Wien's displacement law.

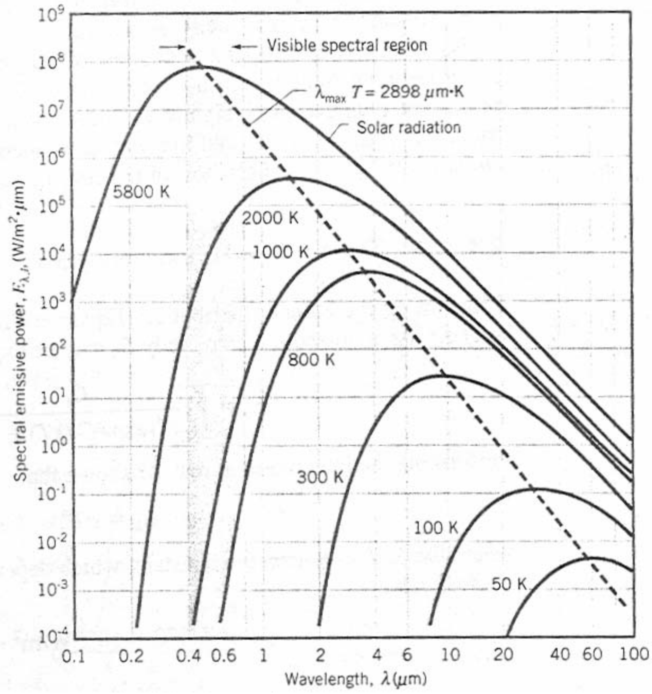


Figure 10.3 Spectral emissive power $E_{\lambda,b}$ as a function of temperature and wavelength

10.3 Radiation Properties of Surfaces

When radiant energy strikes a material surface as shown in Figure 10.4, part of this radiation is reflected, part is absorbed, and part is transmitted. So that radiation properties are defined:

Reflectivity $\rho = \dfrac{\text{reflected part of incoming radiation}}{\text{total incoming radiation}}$

Absorptivity $\alpha = \dfrac{\text{absorbed part of incoming radiation}}{\text{total incoming radiation}}$

Transmissivity $\tau = \dfrac{\text{transmitted part of incoming radiation}}{\text{total incoming radiation}}$

Emissivity $\varepsilon = \dfrac{\text{energy emitted from a surface}}{\text{energy emitted by a black surface at same temperature}}$

The emissivities of clean metallic materials are generally very low while the emissivities of nonmetallic materials are much higher. Typical emissivity values for various materials are shown in Table 10.1. That emissivity is a surface phenomenon can be demonstrated by placing a thin coat of paint on a highly polished metal surface. Before painting a polished copper surface, the emissivity will be about 0.03. However, with just a thin coat of lacquer, 0.00127 cm thick, the emissivity of the same surface will increase sharply to about 0.8.

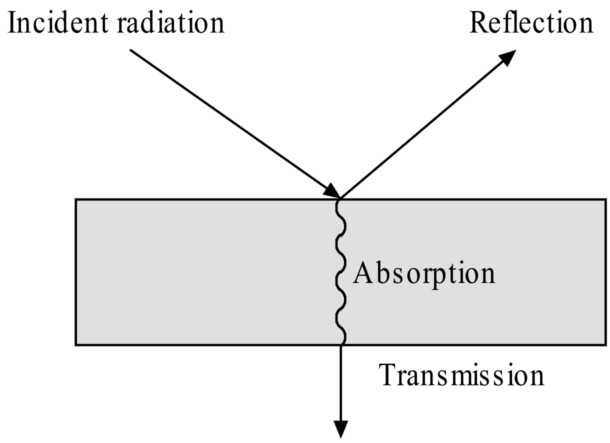


Figure 10.4 Reflection, absorption, and transmission

Since all radiation striking a body must be reflected, absorbed, or transmitted, it follows that

$$\rho \square \square \square 1 \qquad (10.4)$$

In some practical applications surface layers are thick enough to be opaque then the transmissivity may be taken as zero.

All four properties may be functions of wavelength, temperature, incoming direction (except emissivity), and outgoing direction (except absorptivity).

Table 10.1 Typical emissivities at 100 ^{o}C

Material	Emissivity
Aluminum	
Commercial sheet	0.09
Rough polish	0.07
Gold, highly polished	0.018-0.035
Steel, polished	0.06
Iron, polished	0. 14-0.38
Cast iron, machine cut	0.44
Brass, polished	0.06
Copper, polished	0.023-0.052
Polished steel casting	0.52-0.56
Glass, smooth	0.85-0.95
Aluminum oxide	0.33
Anodized aluminum	0.81
Black shiny lacquer on iron	0.8
Black or white lacquer	0.8-0.95
Aluminum paint and lacquer	0.52
Rubber, hard or soft	0.86-0.94

Two types of reflection phenomena may be observed when radiation strikes a surface. The reflection is called specular if the angle of incidence is equal to the angle of reflection as shown in Figure 10.5(a), the reflection is called diffuse when the incident beam is distributed uniformly in all directions after reflection as shown in Figure 10.5(b).No real surface is either specular or diffuse. An ordinary mirror is quite specular for visible light, but would not necessarily be specular over the entire wave length range of thermal radiation. Ordinarily, a rough surface exhibits diffuse behavior better than polished surface. Similarly, a polished surface is more specular than a rough surface.

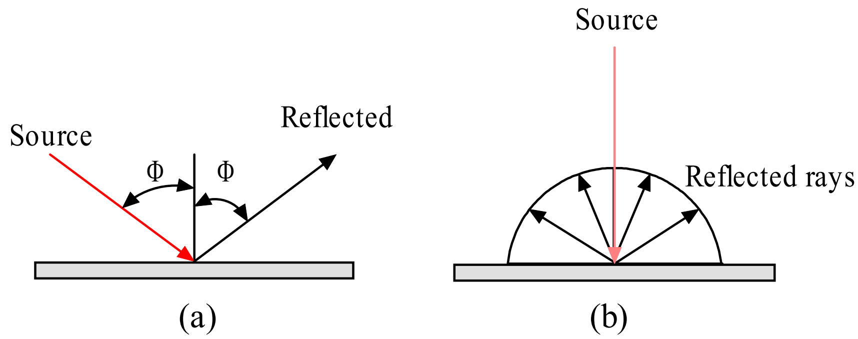


Figure 10.5 (a) Specular reflection (b) Diffuse reflection

10.4 Kirchhoff's Law

Assume that a perfectly black enclosure is available, i.e., one which absorbs all the incident radiation falling upon it, as shown in Figure10.6.

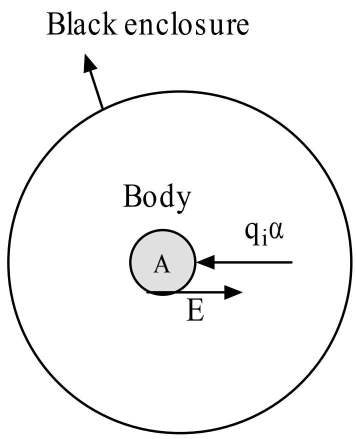


Figure10.6 Model used for deriving Kirchhoff's law

This enclosure will also emit radiation according to the T^4 law. The radiant flux arriving at a given area inside the enclosure is q_i W/m^2.

Now suppose that a body is placed inside the enclosure until it reaches equilibrium. At equilibrium the energy absorbed must be equal to the energy emitted.

The radiation absorbed by the body = $q_i\alpha$ A

The radiation emitted from the body = EA

At equilibrium

$$EA = q_i\alpha A \tag{10.5}$$

If the body is replaced by a black body (α =1) with the same area and shape and allowed to equilibrium at the same temperature.

$$E_b A = q_i A \tag{10.6}$$

If the Equation 10.5 is divided by Equation 10.6 it yields

$$\alpha = E / E_b$$

This ratio is the emissive power of a body to the emissive power of a black body at same temperature. It is also defined before as the emissivity(ε) of the body.

So that, for any surface inside enclosure

$$\alpha = E / E_b = \varepsilon \qquad (10.7)$$

The Equation 10.7 is called Kirchhoff's law.

10.5 Radiation between Black Surfaces and Shape Factors

The shape factor F_{ij} is the fraction of radiant energy leaving one surface that is intercepted by another surface; other names for the shape factor are view factor, angle factor, and geometrical factor. To develop a general expression for F_{ij}, consider two black surfaces has A_1 and A_2 as shown in Figure 10.7.When the two surfaces are maintained at different temperature we want to determine the amount of energy which leaves one surface to another. Firstly we define the shape factor as

F_{1-2} = Fraction of radiation leaving A_1 arriving at A_2

F_{2-1} = Fraction of radiation leaving A_2 arriving at A_1

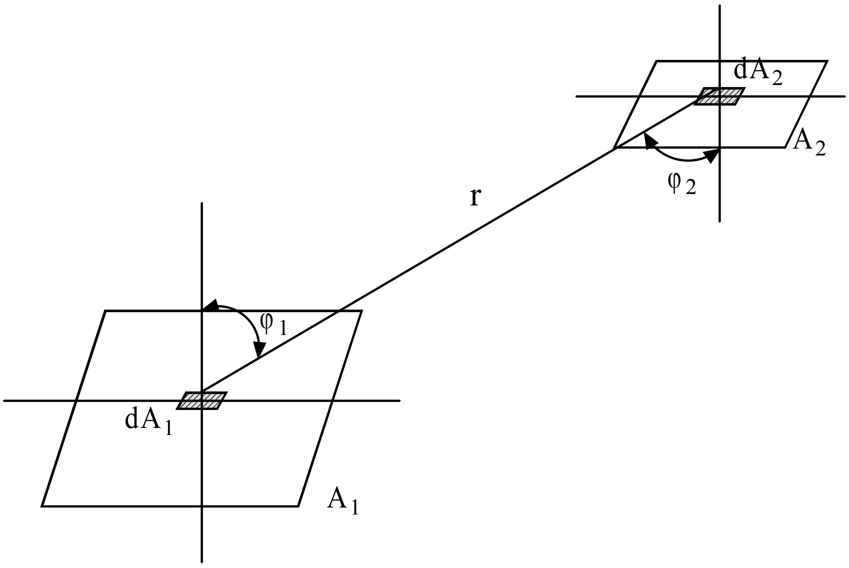


Figure 10.7 Model for deriving radiation shape factor

The energy leaving surface 1 and arriving at surface2 is

$$q_{1-2} = A_1 F_{12}\, E_{b1}$$

The energy leaving surface 2 and arriving at surface1 is

$$q_{2-1} = A_2 F_{21}\, E_{b2}$$

The net energy exchange is

$$q_{net} = A_1 F_{12}\, E_{b1} - A_2 F_{21}\, E_{b2} \qquad (10.8)$$

If the two surfaces are at same temperatures, there would not be heat exchange between them so that q_{net} equals to zero. In this case Equation10.8 will be

$$A_1 F_{12}\, E_{b1} = A_2 F_{21} E_{b2} \qquad (10.9)$$

Also at same temperatures: $E_{b1} = E_{b2}$ which simplify Equation10.9 to

$$A_1 F_{12} = A_2 F_{21} \qquad (10.10)$$

For general expression for any two surfaces i and j:

(10.11)

$$A_iF_{ij} = A_jF_{ji}$$

Equation 10.11 is known as the reciprocity relation

Considering the elements dA_1 and dA_2 as shown in Figure 10.7, the energy leaving surface dA_1 and arriving at surface dA_2 is

$$dq_{1-2} = \frac{E_{b1}}{\pi} dA_1 \cos\varphi_1 \, dw \qquad (10.12)$$

Where dw is the solid angle subtended by dA_2 when viewed from dA_1, its mathematical expression is:

$$dw = \frac{\text{Projection of } dA_2 \text{ on the line between centers}}{r^2} = \frac{dA_2 \cos\varphi_2}{r^2}$$

Substituting in Equation 10.12 gives

$$dq_{1-2} = \frac{E_{b1} \cos\varphi_1 \cos\varphi_2}{\pi r^2} dA_1 dA_2$$

Similarly the energy leaving surface dA_2 and arriving at surface dA_1 is

$$dq_{2-1} = \frac{E_{b2} \cos\varphi_2 \cos\varphi_1}{\pi r^2} dA_1 dA_2 \qquad (10.13)$$

The net energy exchange between dA_1 and dA_2 is

$$\begin{aligned} dq_{net} &= dq_{1-2} - dq_{2-1} \\ &= \frac{(E_{b1} - E_{b2})}{\pi r^2} \cos\varphi_2 \cos\varphi_1 dA_1 dA_2 \end{aligned} \qquad (10.14)$$

Performing double integration on Equation 10.14 with respect to A_1 and A_2 yields

$$q_{net} = (E_{b1} - E_{b2}) \iint_{A_2\, A_1} \frac{\cos\varphi_2 \cos\varphi_1 dA_1 dA_2}{\pi r^2} \qquad (10.15)$$

But

$$A_1F_{12} = A_2F_{21} = \iint_{A_2\, A_1} \frac{\cos\varphi_2 \cos\varphi_1 dA_1 dA_2}{\pi r^2} \qquad (10.16)$$

This gives

$$\begin{aligned} q_{net} &= (E_{b1} - E_{b2}) A_1 F_{12} \\ &= (E_{b1} - E_{b2}) A_2 F_{21} \\ &= \sigma (T_1^4 - T_2^4) A_2 F_{21} \end{aligned} \qquad (10.17)$$

A few graphical results that present the shape factor for important geometries are shown in Appendix A.

But For nontrivial geometries shape factors must be calculated by double integration of Equation10.16.

10.6 Shape Factor Relations

Some important shape factor relations are suggested below.

10.6.1 Reciprocity Relation

As explained before during previous derivations, this relation is useful in determining one shape factor from knowledge of the other.

$$A_iF_{ij} = A_jF_{ji}$$

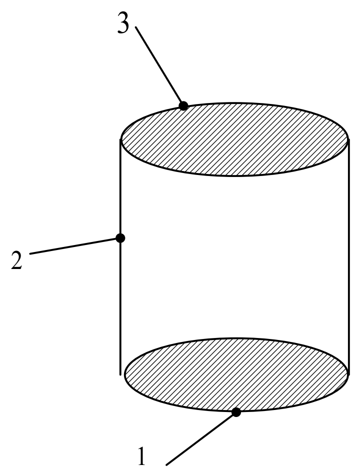
10.6.2 Summation Rule

$$\sum_{j=1}^{N} F_{ij} = 1 \qquad (10.18)$$

Note that the term F_{ii} is non zero if the surface is concave because it sees it self and equal zero for a plane or convex surface.

This role is applied on enclosures surfaces.

Example 10.1: Determine the view factor F_{12} for the Figure shown below knowing that $D_1 = D_2 = 5$ cm and a = 8 cm.



Solution:

For an enclosure $$\sum_{j=1}^{N} F_{ij} = F_{11} + F_{12} + F_{13} = 1$$

Where $F_{11} = 0$

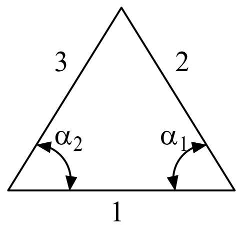
From Figure A.1 at $r_1/a = r_2/a = 5/8 = 0.625$

$$\therefore\ F_{13} = 0.22$$

Then

$$F_{12} = 1 - F_{13} = 1 - 0.22 = 0.78$$

Example 10.2: Determine the view factor F_{12} and F_{13} for the figure shown below knowing that the angles $\alpha_1 = \alpha_2$.



Solution:
It is an enclosure rectangle so that

$$F_{11} + F_{12} + F_{13} = 1$$

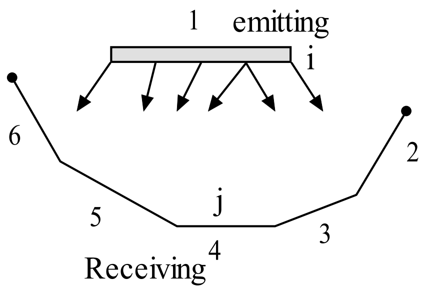
Where $F_{11} = 0$
Because $\alpha_1 = \alpha_2$ then

$$F_{12} = F_{13} = 0.5$$

10.6.3 Division of Receiving Surface

$$F_{ij} = \sum_{k=1}^{n} F_{ik} \qquad (10.19)$$

I.e. chip exposed to several heat sinks as shown below



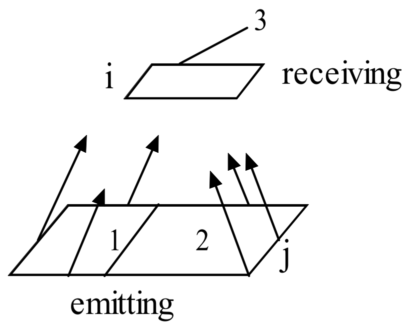
The fraction of heat radiates from chip 1 arrived to the another surfaces2, 3,4,5,6 is

$$F_{1-2,\,3,4,5,6} = F_{12} + F_{13} + F_{14} + F_{15} + F_{16}$$

10.6.4 Division of Emitting Surface

$$A_j F_{ji} = \sum_{k=1}^{n} A_k F_{ki} \qquad (10.20)$$

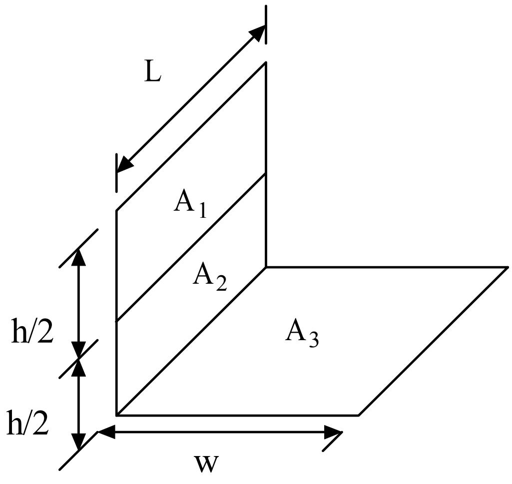
The schematic shown below is a typical example for Division of emitting surface.



$$A_j = \sum_{k=1}^{n} A_k$$

Where $A_{1,2} F_{1,2-3} = A_1 F_{13} + A_2 F_{23}$

Example 10.3: Determine the view factor F_{13} for the Figure shown below knowing that it's dimensions $L = h = w = 1$ cm.



Solution:
By taking the areas $A_1$and A_2 with A_3 and using Equation 10.19 to gives

$$F_{3-1,2} = F_{31} + F_{32}$$

From Figure A.3 at
W = 1 and H = 1

$$\therefore F_{3-1,2} = 0.2$$

By taking the areas A_2 with A_3 and from Figure A.3 at
H = 0.5 and W = 1

$$\therefore F_{32} = 0.15$$

Substitute in main equation to gives

$$F_{31} = 0.2 - 0.15 = 0.05$$

By using reciprocity relation
$A_3F_{31}=A_1F_{13}$
Then $\therefore F_{13} = 0.1$

Appendix A

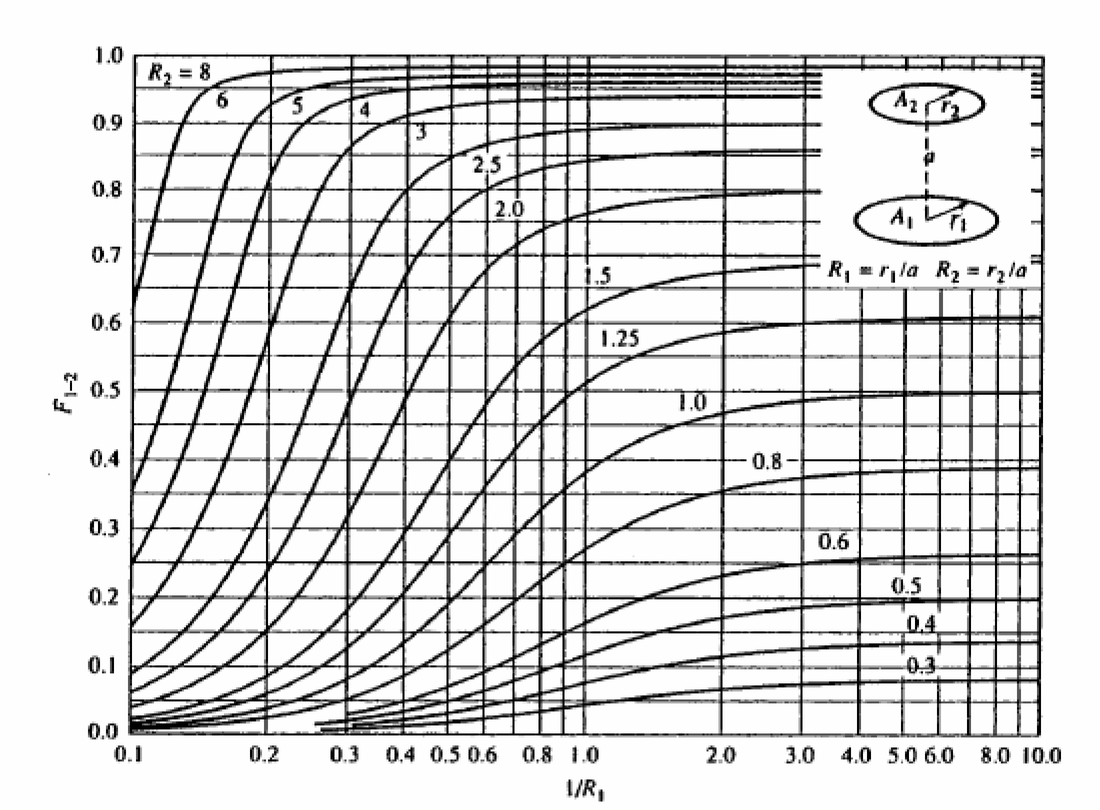


Figure A.1 View factor between parallel, coaxial disks of unequal radius

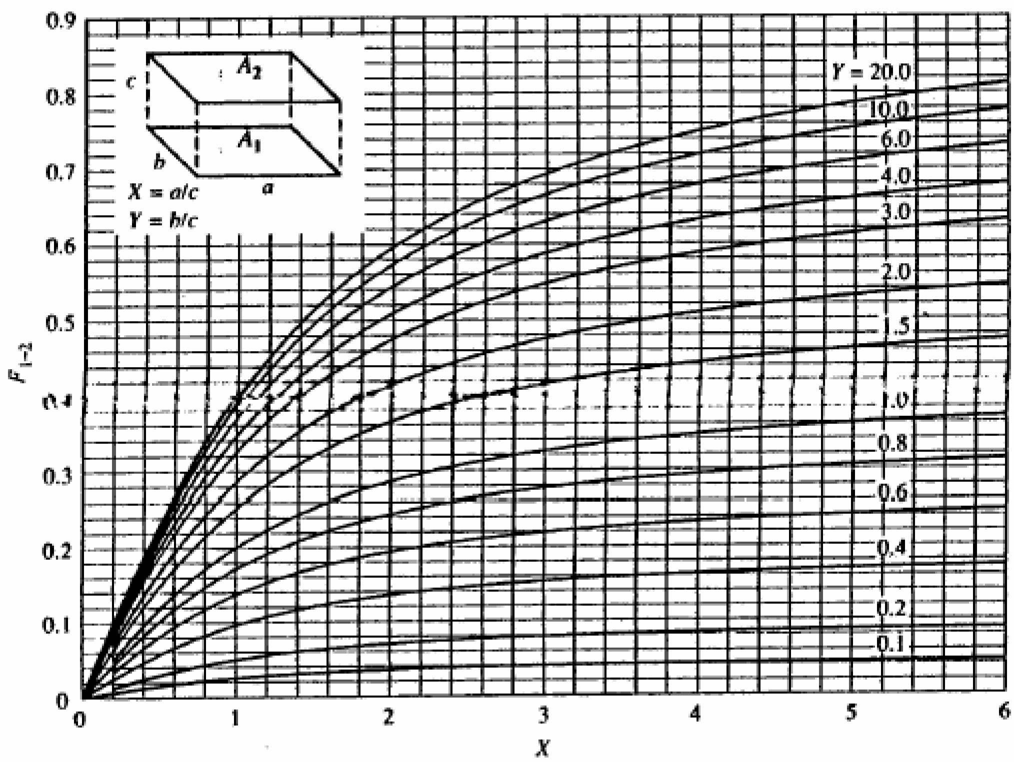


Figure A.2 View factor between identical, parallel, directly opposed rectangles

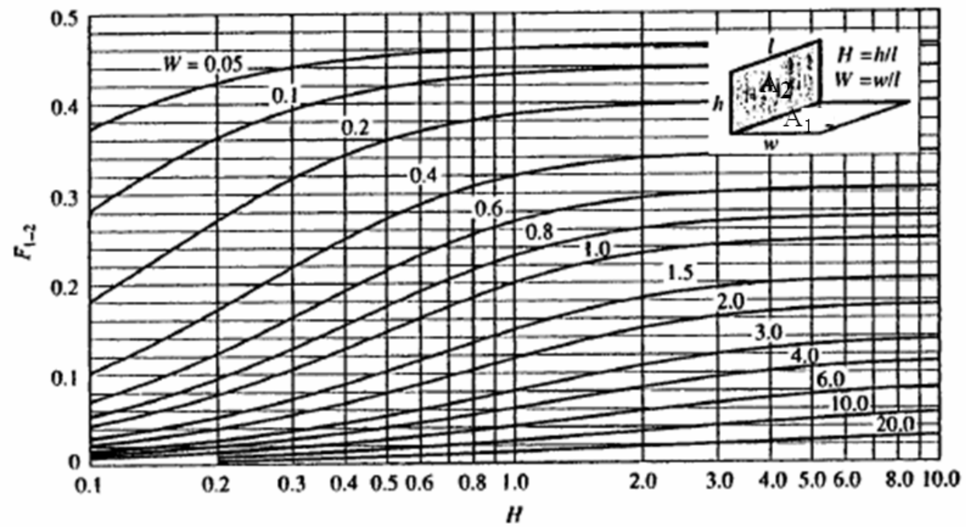


Figure A.3 View factor between perpendicular rectangles with common edge